Competing Openly or Blindly in Crowdsourcing Contests?

Lian Jian, Zheng Li, and Tracy Xiao Liu*

July 24, 2014

Abstract

Organizations are increasingly outsourcing tasks once performed in-house to wider participants on the Internet by hosting online contests. In practice, two types of mechanisms are used to organize these contests: simultaneous (blind) and sequential (open). In a simultaneous contest, contestants submit their solutions independently without access to one another's submissions, while in a sequential contest, contestants submit their solutions sequentially and each can view all prior submissions before making their decisions. Most prior theoretical and experimental research has focused on simultaneous contests, with only a handful that have studied sequential ones. In this paper, under the condition of incomplete information, we analytically show that simultaneous contests produce higher quality best solutions than sequential contests. Using a laboratory experiment, we test this theoretical prediction as well as the prediction that simultaneous contests are more efficient than sequential contests. Our data support both predictions. We also discover that as the number of contestants increases, the efficiency of sequential contests drops significantly, further reducing their performance relative to simultaneous contests.

Keywords: sequential contest, experiment, crowdsourcing
JEL Classification: C7, C91

*Jian: Annenberg School for Communication, University of Southern California, Los Angeles, CA, U.S.A. Email: ljian@usc.edu. Li: Department of Economics, School of Economics and Management, Tsinghua University, Beijing, China. Email: lizh4.12@sem.tsinghua.edu.cn. Liu: Department of Economics, School of Economics and Management, Tsinghua University, Beijing, China. Email: tracy.umich@gmail.com.

1 Introduction

Contests have been previously shown as effective mechanisms for eliciting individual effort in workplace (Amegashie et al., 2007; Che and Gale, 2003; Dechenaux et al., 2014), for solving challenging problems for governments (e.g., the DARPA Grand Challenge by the United States Department of Defense) and large organizations (e.g., the Netflix Prize). Naturally, some contests, e.g., rent-seeking (Tullock, 1980), can be modeled as simultaneous games, since the contestants choose their effort independently and simultaneously. Some other contests are better modeled as sequential games, e.g., litigations (Morgan, 2003), since the participants enter the contests at different times and late contestants make their effort choices after observing prior contestants' solutions.

While a contest holder in the aforementioned cases often has a limited choice in the format (simultaneous versus sequential) of the contest due to the nature of their problems, the recent popularity of online contests has made the contest format a clear decision variable. These contests, often called crowdsourcing contests, are increasingly used by businesses and non-profit organizations to outsource tasks once performed in-house to large pools of participants on the Internet, to seek solutions for new product innovation (e.g., InnoCentive.com), algorithm design (Topcoder.com), graphic design (e.g., Crowdspring.com), as well as to perform routine tasks such as translation and programming (e.g., Taskcn.com). In turn, online platforms that facilitate such activities have emerged as a rapidly growing industry. In 2011, 14 major crowdsourcing firms earned $50 million in total, which was a 74% increase from 2010, and 53% from a year earlier (Silverman, 2012).

On these online contest platforms, both simultaneous and sequential contests have been implemented. For example, on Taskcn.com (Liu et al., 2014), contests are sequential (or, open)[1] — and on InnoCentive.com (Jeppesen and Lakhani, 2010), contests have been implemented as simultaneous (or, blind). On yet another platform, 99designs.com, contest holders get to choose whether they want their contests to be open or blind (Wooten and Ulrich, 2013). And in practice, most contest holders choose open.[2]

Little research exists to guide the choice between these two contest mechanisms. Even though some platforms, such as 99designs, recommend blind contests over open contests for attracting higher quality solutions,[3] few systematic evaluations of these two mechanisms exist to substantiate such claims, except for three recent experimental studies (Boudreau and Lakhani, 2014; Liu, 2013;

[1]Intellectual property concerns naturally arise in open contests. To mitigate such concerns, popular crowdsourcing sites, such as 99designs.com and Logomyway.com, implement easy-to-use web interfaces for reporting and dealing with plagiarism (Bockstedt et al., 2014). Further, empirical evidence has shown that in open innovation, actual concerns of plagiarism seem much lower than expected and are outweighed by the benefits gained from open communication (Harhoff et al., 2003; von Hippel, 2005). In fact, in open logo design contests on Logomyway.com, participants who make their first submissions earlier, in spite of higher risks of intellectual property loss, are more likely to win (Bockstedt et al., 2014).

[2]Out of the first 100 reverse-chronologically listed logo-design contests on Sept 3, 2013, only six were blind and the remaining 94 contests were all open. Observations were made at 10am PST on Sept 3, 2013 on this page: `http://99designs.com/logo-design/contests?show=open&page=1`. Records are available from the authors upon request.

[3]See https://99designs.com/help/contesttypes, retrieved on March 15, 2014.

Wooten and Ulrich, 2013, more on these studies later). In fact, the majority of the existing theoretical and experimental literature has focused on simultaneous contests, while only a handful have studied sequential contests, leaving many properties of sequential contests unexplored, let alone the relative performance of the two mechanisms (see Dechenaux et al., 2014, for a review).[4] This could be due to the challenges in solving for the equilibria in sequential contests: to the best of our knowledge, the only explicit solution of Bayesian Nash Equilibrium (BNE) has recently been provided by Segev and Sela (2014a) for a specific family of distribution functions for participants' abilities.

We follow prior literature (Chawla et al., 2012; DiPalantino and Vojnovic, 2009; Terwiesch and Xu, 2008) to use all-pay auctions — auctions in which all bidders pay their bids regardless of winning and the highest bidder wins — to model crowdsourcing contests. We also keep in mind that in crowdsourcing, a contest holder profits primarily from the quality of the best solution, rather than from the sum of all solutions' qualities, or bids. Furthermore, we study the two types of contests under incomplete information, due to considerations that many participants in online contests have limited information about one another (DiPalantino and Vojnovic, 2009). We shall clarify that our definition of "sequential" contests is different from sequential open cry auctions, in particular because in our sequential contests, each participant only makes one decision while in sequential open cry auctions each participant has unlimited opportunities to bid.

In this paper, building on work by Segev and Sela (2014a), who derive the BNE bidding function of n-player sequential all-pay auctions under the assumption of asymmetric bidders (bidders with potentially different prior value distributions), we characterize the individual performance of symmetric participants in sequential contests and derive the explicit solution for the expected highest quality obtained in such contests. More importantly, we compare the expected highest quality generated by sequential and simultaneous contests. We also conduct a laboratory experiment to compare these two contests and to study the effect of increased contest sizes.

To the best of our knowledge, this is the first study to analytically compare simultaneous and sequential contests under incomplete information and to experimentally test the theoretical predictions. Both our analytical and experimental results show a lopsided advantage of simultaneous contests over sequential contests. Simultaneous contests induce higher quality best solutions and produce more efficient outcomes. Furthermore, while it might be intuitive to expect sequential contests to waste less participant effort since contestants in these contests can coordinate their effort, using a special case, we show that this is not necessarily true: simultaneous contests can be less wasteful than sequential contests. Indeed, our experimental data also support this result. Moreover, we discover that as the number of contestants increases, efficiency decreases in sequential contests, further deteriorating their performance relative to simultaneous contests. Additionally, we find that in simultaneous contests, when low-ability contestants greatly outnumber high-ability ones, high-ability contestants exert significantly lower effort than their BNE predictions. We ex-

[4] Although many recent empirical studies based on observational field data have explored various characteristics of both simultaneous (e.g., Boudreau et al., 2011) and sequential contests (e.g., Bockstedt et al., 2014) separately, again, none has compared the two mechanisms explicitly.

plain this phenomenon with a model of overconfidence in which high-ability players overestimate their winning probabilities and consequently choose to under-invest their efforts.

2 Literature Review

Focusing on contests modeled as all-pay auctions, below, we start our literature review with studies on simultaneous contests and all-pay auctions and then discuss studies on sequential contests, followed by empirical findings both in the laboratory and in the field. As mentioned earlier, our study treats these contests as games of incomplete information. Therefore, unless otherwise noted, our review of the literature focuses on studies that assume incomplete information, and refer our reader to Dechenaux et al. (2014) and Konrad (2009) for comprehensive reviews of the theoretical and experimental literature on contests and all-pay auctions with complete information. We should also note that the all-pay auction literature is related to a few other types of auction mechanisms such as bucket auctions (Carpenter et al., 2011) and penny auctions (Augenblick, 2014). Additionally, there is a growing literature which comparatively examines the performance of different contests, e.g, all-pay auction vs. lottery contest (Duffy and Matros, 2013; Sheremeta et al., 2010).

2.1 Theoretical literature

Our model of simultaneous crowdsourcing contests closely follows the extant literature on this topic, which has restricted its attention to games with symmetric participants, or bidders. That is, all bidders are assumed to be a priori identical and share common prior beliefs about bidders' type distribution. [5] Assuming *i.i.d.* distributions of bidder types, Weber (1985) and Hillman and Riley (1989) derive a monotonic (i.e., bids monotonically increase in bidders' private values) symmetric equilibrium and provide the explicit expressions for the equilibrium bidding strategies. Further, Chawla and Hartline (2013) establish the uniqueness of this equilibrium.

Beyond the simple model of all-pay auctions, a number of studies build models that highlight various features of crowdsourcing contests. Terwiesch and Xu (2008), in particular, characterize the problem space of crowdsourced tasks and categorize them as expertise-based (where solvers' expertise and effort determine the quality of their submissions), ideation (where seekers' taste is highly subjective), and trial-and-error (where the success of a solution is highly uncertain) tasks. Our study falls under the expertise-based task case since we assume participants' ability and effort completely determine the quality of their solutions. A typical example of such a task is a translation task for which contestants' higher expertise and effort lead to higher quality with little noise and the best solution can be unambiguously identified (Liu et al., 2014).

Equilibrium effort can increase or decrease with respect to n, the number of participants, depending on the interaction of two effects. On one hand, there is a "negative incentive effect"

[5] There are a number of notable exceptions. For example, Amann and Leininger (1996), Parreiras and Rubinchik (2010), Kirkegaard (2012), and Segev and Sela (2014a; 2014b; 2014c) examine equilibrium behaviors in contests with *ex ante* asymmetric bidders (e.g., their private types follow different prior probability distributions).

previously suggested in analyses of contests with both complete information (Che and Gale, 2003; Fullerton and McAfee, 1999; Taylor, 1995) and incomplete information (e.g., Archak and Sundararajan, 2009; Moldovanu and Sela, 2006; Terwiesch and Xu, 2008). That is, as n increases, one's probability of winning decreases, leading her to make a lower effort to avoid loss. On the other hand, there exists a "positive incentive effect" as well (Archak and Sundararajan, 2009; Moldovanu and Sela, 2006). With a relatively small n, high ability participants would indeed increase their equilibrium effort as n increases, because these participants can take advantage of their low participation cost and would find that with an increased n, their extra effort achieves higher "bang for the buck" by beating more contestants.[6] These two effects together shape the monotonicity of the equilibrium effort function with respect to n, and which of these effects will dominate depends on the particular distribution of participant abilities (Archak and Sundararajan, 2009; Moldovanu and Sela, 2006). In particular, Moldovanu and Sela (2006) show that increasing n will have a positive effect on high ability players but a negative effect on low ability players.

In addition to the positive and negative incentive effects, Terwiesch and Xu (2008) highlight a "parallel path" effect (Dahan and Mendelson, 2001), and argue that even though a higher n could lead to a lower equilibrium effort in their model, a larger pool of participants can also lead to a higher chance of discovering talents. They identify conditions about the ability distributions that can lead to a situation in which the parallel path effect dominates the negative incentive effect, thereby leading to a higher quality best solution. In general, the expected highest quality is not monotonic in n (Archak and Sundararajan, 2009; Moldovanu and Sela, 2006) but asymptotically, it converges to 1/2 in single-winner contests (Archak and Sundararajan, 2009; Chawla et al., 2012; Moldovanu and Sela, 2006).

Besides characterizing equilibrium properties, prior research has explored various design factors to find the optimal contest formats. Under fairly general conditions, single-winner contests (the contest format that we adopt) have been shown to be optimal either for maximizing the quality of the best solution (Chawla et al., 2012) or for maximizing a contest holder's profit, i.e., the sum of qualities of top solutions received minus the prizes paid out (Archak and Sundararajan, 2009).[7] Such contests are also found to be optimal when maximizing the total effort is the goal (Glazer and Hussin, 1988; Moldovanu and Sela, 2001). Moldovanu and Sela (2006) consider the optimal number of contest stages and the number of rewards and find that to maximize total effort from all participants, a single-stage single-winner contest is optimal but to maximize the highest effort, it is best to split the competitors into two divisions and have a second stage in which divisional winners compete for a single prize. Although we recognize the advantage of splitting the contest into divisions, here, we choose to focus on single-division contests for their simplicity.

In contrast to the large volume of studies on simultaneous all-pay auctions and contests, few

[6] The positive incentive effect does not exist in Terwiesch and Xu's (2008) model, because the marginal cost of effort is modeled as a constant among all contestants as opposed to a function of one's private ability (Archak and Sundararajan, 2009; Chawla et al., 2012; Hillman and Riley, 1989; Moldovanu and Sela, 2006; Weber, 1985). We follow the latter modeling approach and thus will permit both negative and positive incentive effects in our model.

[7] When contestants are risk-averse, multiple prizes can be optimal for maximizing profit (Archak and Sundararajan, 2009).

studies have examined sequential ones. Our study closely follows the line of work pioneered by Segev and Sela (2014a; 2014b; 2014c), who study sequential contests under incomplete-information conditions. In this setting, the number of stages equals the number of participants, and participants are assumed to be asymmetric, i.e., having different (but commonly known) ability distributions. Focusing on two-player contests with general ability distribution functions, Segev and Sela explore effects of giving the early player a head start (2014b) and adding random noise into players' outputs (2014c). Focusing on a more general model with n players, Segev and Sela (2014a) find that, among other things, the expected highest quality is not monotonic in the number of contestants in general. They also identify another counterintuitive result which predicts that n weak players might produce higher best quality solutions than n stronger players. Further, with an added assumption that all contestants' abilities follow a family of power distribution functions ($F_i(x) = x^{c_i}, 0 < c_i < 1, i \in 1, \cdots, n$), Segev and Sela (2014a) derive the closed-form solution for the BNE effort functions, which we utilize in our study. Under this specific assumption, if all participants' abilities are *i.i.d* distributed (i.e., with the same parameter $c_i = c$), they show that the expected highest quality monotonically increases in n.

The unique contribution of this study to the body of theoretical literature on contests is that we identify a real-life situation (i.e., online contests) in which a choice between simultaneous and sequential contests has to be made, and theoretically analyze the relative performance of these two contest mechanisms. To the best of our knowledge, this is the first study to systematically compare these two contest mechanisms and show that with symmetric contestants, simultaneous contests perform better than sequential contests in eliciting higher quality best solutions, higher average quality solutions and in producing more efficient outcomes.

2.2 Empirical literature

In this section, we review laboratory and field experimental studies as well as studies based on field data. These studies have tested the theoretical predictions as well as explored various factors influencing contest outcomes.

Compared to theoretical predictions that assume risk-neutral contestants, prior experimental studies of incomplete information simultaneous all-pay auctions have reported that high ability individuals overbid and low ability ones underbid (Barut et al., 2002; Muller and Schotter, 2010; Noussair and Silver, 2006). An overbidding phenomenon is also observed in complete information all-pay auctions (Davis and Reilly, 1998; Gneezy and Smorodinsky, 2006). Typically, these experiments employ relatively large contest sizes, e.g., $n = 6$ (Noussair and Silver, 2006), and use uniform distribution of ability factors if the game is studied under incomplete information. In a few experiments with small contest sizes, e.g., $n = 2$ (Grosskopf et al., 2010; Potters et al., 1998), however, no significant over- or under-bidding behavior has been observed. Consequently, Dechenaux et al. (2014) conclude that whether overbidding can be observed is parameter-dependent and the findings from large all-pay auctions may not hold in smaller auctions.

With respect to the effects of contest size on the quality of best solutions, all three effects predicted by theory — the negative and positive incentive effects as well as the parallel path effect — have found empirical support. The negative incentive effect has been consistently demonstrated by both experimental and field studies. For example, although no prior research directly examines the size effect under incomplete information all-pay auctions, a laboratory experiment conducted by Gneezy and Smorodinsky (2006) has found that increasing the number of bidders decreases the average effort in complete-information all-pay auctions with symmetric bidders. Similar evidence for the negative incentive effect has been found in studies using data from crowdsourcing platforms, such as TopCoder.com (Boudreau et al., 2011, 2013) and Threadless.com (Huang et al., 2014). The positive incentive effect has been identified by Boudreau et al. (2013) using data from TopCoder.com. They find that while low ability participants respond to increased competition negatively, the presence of superstars actually induces higher effort from highly skilled participants. Last, the parallel path effect has been reported by multiple studies (Araujo, 2013; Boudreau et al., 2011). For example, Boudreau et al. (2011) find that when the task involves a sufficient level of uncertainty, the parallel path effect will dominate the negative incentive effect such that higher numbers of participants lead to higher chances of innovation.

With regard to contest format, the focus of our central research question, most extant empirical research has focused on one type of contest, either simultaneous (e.g., InnoCentive.com and TopCoder.com), sequential (e.g., 99designs.com, Logomyway.com, and Crowdspring.com), or a combination or hybrid of the two (e.g., Taskcn.com, 99designs.com and Crowdspring.com). From a contest holder's perspective, these studies have examined the effects of contest characteristics on their outcomes, such as the reward size (Araujo, 2013; Huang et al., 2014; Walter and Back, 2011; J. Yang et al., 2008; Y. Yang et al., 2009), contest duration (Yang et al., 2009), the level of competition (Boudreau et al., 2011, 2013), the brand-strength of the contest holder (Walter and Back, 2011), whether the contest holder provides in-process feedback to the contestants (Wooten and Ulrich, 2011; Yang et al., 2009) and whether a reserve (a high quality solution) already exists (Liu et al., 2014). Other studies have taken the contestants' perspective, studying their motivations (Brabham, 2010) and trying to predict their chances of winning (Archak and Ghose, 2010; Bockstedt et al., 2014; Jeppesen and Lakhani, 2010; J. Yang et al., 2008; Y. Yang et al., 2011).

Three studies are most relevant to our study as they explicitly compare the performance of simultaneous and sequential contests. Of these, Liu (2013) analytically and experimentally compares the two mechanisms under complete information (our study models these contests under incomplete information), while allowing the participants to choose between sequential and simultaneous contests. She finds that simultaneous all-pay auctions produce higher revenue (sum of all bids) than sequential auctions. In addition, two recent field experiments have compared the two contest mechanisms by running logo design contests on 99designs.com and Crowdspring.com (Wooten and Ulrich, 2013) and by running bioinformatics innovation contests on TopCoder.com (Boudreau and Lakhani, 2014). Wooten and Ulrich (2013) find that simultaneous contests attract fewer participants but enable high ability participants to produce higher quality solutions. Boudreau and Lakhani's (2014) experiment finds that simultaneous contests incentivize their participants to make higher efforts but do not necessarily lead to higher quality solutions, primarily because sequential contests help their contestants save effort by focusing on the most promising techniques.

To the best of our knowledge, no prior research has analytically compared the expected highest submission quality in simultaneous and sequential contests under incomplete information. This study attempts to make a first step to fill this gap, as well as to use laboratory experiments to test the theoretical predictions. Furthermore, compared to prior research which assumes that contestants' ability factors are drawn from uniform distributions, our study compares these two mechanisms under a skewed power distribution function, which is arguably more realistic for modeling crowdsourcing markets (Araujo, 2013; Yang et al., 2008). This is also the first laboratory experimental study to investigate the size effect on individual behavior for both sequential and simultaneous contests under incomplete information.

3 Theoretical Analysis

A single task is to be crowdsourced through a contest modeled as an all-pay auction. That is, regardless of who wins the contest, all contestants incur the cost of their submissions, typically in time or effort. An indivisible prize worth $v = 1$ is to be awarded to a single winner who submits the solution with the highest quality. We assume that each participant's effort unambiguously determines the quality of the submission, and that the quality is objectively quantifiable. Therefore, we use the words "bid," "quality," and "effort" interchangeably throughout. We also use "participant," "contestant," and "player" as synonyms.

There are n contestants who vary in ability. Let $a_i \geq 0$ be contestant i's ability factor, which is her private information. Contestants' ability factors are *i.i.d.* draws from the interval $[0, 1]$ according to a cumulative distribution function, $F(x)$, which is common knowledge. We assume that $F(x) = x^c$, where $c \in (0, 1)$, which is the most general class of CDFs for which we know the explicit analytical solutions to the BNE of sequential contests, due to Segev and Sela (2014a), who have studied sequential contests with asymmetric contestants, i.e., each contestant can have a different parameter c.

The parameter c, which we call the "talent" parameter, affects the relative proportion of low and high ability contestants. As illustrated in Figure 1, as c increases, the proportion of high ability contestants increases and the proportion of low ability contestants decreases. As c approaches 1, the distribution approaches a uniform distribution. For any $c \in (0, 1)$, there are many more low ability contestants than high ability ones, which is usually the case in many online crowdsourcing markets (Araujo, 2013; Yang et al., 2008).

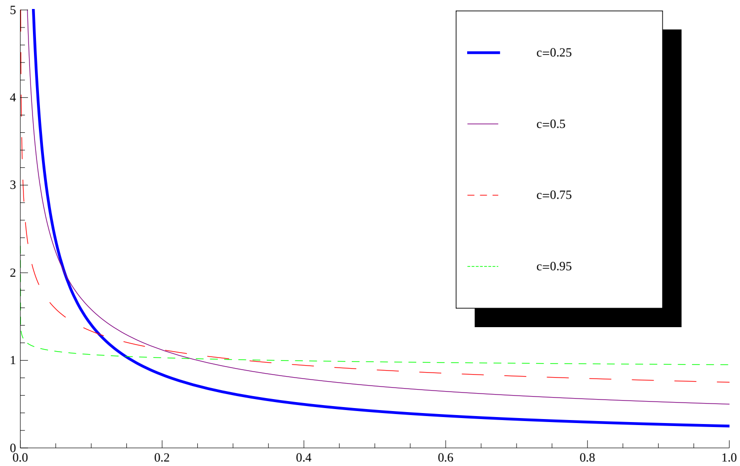


Figure 1: The probability density function of ability factors with different values for parameter c.

For contestant i, a submission of quality q_i costs q_i/a_i, indicating that it is less costly for a high ability contestant to create and submit the same quality solution than a low ability contestant. In sequential contests, we assume that ties are broken in favor of the late entrant, although alternatively one could assume that a late entrant could always win by making an effort just a tiny amount (ϵ) higher than the existing highest.

3.1 Equilibrium characterizations

The unique Bayesian Nash equilibrium effort function in simultaneous contests has been derived in prior research (Chawla and Hartline, 2013; Hillman and Riley, 1989; Krishna and Morgan, 1997; Weber, 1985). For completeness, we reproduce the equilibrium effort function according to our model setup in Equation (1):

$$q^{Sim}(a_i) = \frac{(n-1)c}{1+(n-1)c} a_i^{1+(n-1)c} \tag{1}$$

The fact that $q^{Sim}(a_i)$ monotonically increases in a_i implies that in a simultaneous contest the contestant with the highest ability always wins (Archak and Sundararajan, 2009). $q^{Sim}(a_i)$, however, is not monotonic in n or in c. This is understandable given that when either n or c increases, competition intensifies, triggering both the negative and positive incentive effects. The dominant effect of the two determines the monotonicity of the function at any particular point of n and c.

For sequential contests, Segev and Sela (2014a) derive the equilibrium effort function (Equation (2)).

$$q^{Seq}(a_i) = \begin{cases} 0 & \text{if } 0 \leq a_i < \overleftarrow{a}_i \\ \max\{q_j(a_j)\}_{j<i} & \text{if } \overleftarrow{a}_i \leq a_i < \overrightarrow{a}_i, \\ (a_i(1-d_i))^{\frac{1}{d_i}} & \text{if } \overrightarrow{a}_i \leq a_i \leq 1. \end{cases} \tag{2}$$

where $\overleftarrow{a}_i = [\max\{q_j(a_j)\}_{j<i}]^{d_i}$, $\overrightarrow{a}_i = \frac{1}{1-d_i}[\max\{q_j(a_j)\}_{j<i}]^{d_i}$, and $d_i \equiv (1-c)^{n-i}$.

This function is broken into three parts based on the range in which contestant i's ability factor falls, relative to the maximum quality produced by all preceding contestants, i.e., $\max\{q_j(a_j)\}_{j<i}$. In the first part, $\max\{q_j(a_j)\}_{j<i}$ is too large compared to a_i, and thus contestant i's best option is to "fold", i.e., to make zero effort and forgo the opportunity of winning. In the second part, $\max\{q_j(a_j)\}_{j<i}$ is not too large, so contestant i's best option is to play the "call" strategy, i.e., to make an effort equal to $\max\{q_j(a_j)\}_{j<i}$. In the third part, $\max\{q_j(a_j)\}_{j<i}$ is relatively small, so contestant i "raises" by choosing a submission quality, $(a_i(1-d_i))^{\frac{1}{d_i}}$, that is higher than $\max\{q_j(a_j)\}_{j<i}$. This quality is a global maximizer of her expected payoff. Here, we use q_i^* to denote this global maximizer for player i, and further analyze its properties.

In terms of winning, there is a clear late-mover advantage in sequential contests, as Proposition 1 demonstrates. Based on the analytical solution it provides, we compute that in a two-player contest, player 1 has a 27% chance of winning and player 2 has 73%. In a three-player contest, player 1, 2 and 3 have 13%, 25%, and 63% respectively. Apparently, the last mover enjoys a disproportionally higher chance of winning compared to all earlier entrants.

Proposition 1. *The expected winning probability of contestant i, $P(i)$, increases in i ($i \in \{1, \cdots, n\}$). The explicit expression of $P(i)$ is derived in Lemma 1 in Appendix A.*

Proof. See Appendix A. □

Aside from the winner, another important player in a contest is the first one to raise the maximum quality to the final winning level. These two players are not necessarily the same person, since a winner could be playing the call strategy, rather than raise. A contest holder's ultimate concern is the maximum quality she receives from all contestants. The only way the maximum quality level can be raised is through players playing the raise strategy, i.e., q_i^* for player i. Therefore, q_i^* represents the highest potential contribution player i can make to the contest, i.e., by increasing the quality of the best solution. Here, we call q_i^* contestant i's "capacity." Understanding how q_i^* varies with respect to i and a_i could help reveal the internal dynamics of sequential contests. In Proposition 2, we temporarily treat i as a continuous variable and examine how participants' capacity varies with their positions.

Proposition 2. *For any given ability factor $a \in [0,1]$, $q_i^*(a)$ first increases and then decreases in i, and attains its global maximum on $[1, n-1]$. Let $\hat{i}$ be the global maximizer. As the total number of contestants, n, increases, $\hat{i}$ weakly increases; as a increases, $\hat{i}$ weakly decreases; and finally, as the talent level, c, in the participant pool increases, $\hat{i}$ weakly increases.*

Proof. See Appendix B. □

Proposition 2 shows that a player's capacity does not monotonically vary with her position in a contest. Intuitively, i is an indicator of the competition faced by player i (the lower the i the higher the competition). Whether higher competition leads to higher potential effort depends on the current competition level and player i's ability factor. If the competition level is relatively low for player i with a_i, increasing competition will encourage player i to exert higher effort (the positive incentive effect). On the other hand, if the current competition level is already high for player i, she will be discouraged from raising her effort as competition intensifies, since the chance of winning is too slim (the negative incentive effect).

Here, as i increases, player i faces less competition, motivating her to lower her effort and win with a higher surplus (the reversed positive incentive effect). On the flip side, as i increases, her winning probability increases (the reversed negative incentive effect). Further, as i increases, the impact of a marginal increase in i's effort has larger influences on i's winning probability,[8] motivating i to make a higher effort. The interaction of these dynamics results in q_i^* first increasing and then decreasing in i.

Proposition 2 also shows that as the total number of participants increases, the position that maximizes player i's capacity, $\hat{i}$, also moves toward the end, where there is lower competition. However, as a player's ability increases, the capacity-maximizing position moves toward the beginning of the contest, since the high competition faced at the beginning of a contest can stimulate her to make the best effort. Last, as c increases, the number of high ability contestants increases, implying higher competition overall in a contest. As a result, the capacity-maximizing position moves toward the end to offset the increased competition level. Taken together, these results suggest that precisely which position maximizes a player's capacity is dependent on the interaction of a number of factors, including the number of participants, the player's ability, and the overall distribution of abilities among all participants. In general, if high ability participants enter a contest early, medium ability participants enter in the middle, and low ability participants enter late, the contest might garner better solutions than if they entered the other way around: low ability ones early, medium ability next, and high ability ones last. Such dynamics show that the sequence of entry plays an important role in determining the quality of the best solution in a contest, as highlighted by Segev and Sela (2014a) in their analysis of optimal entry order in sequential contests. In our particular setting, given that the order of entry is exogenous and random, these dynamics imply that in expectation, the expected quality of the best solutions in sequential contests might suffer.

In summary, both Proposition 1 and 2 characterize the internal dynamics of sequential contests. From a contestant's perspective, Proposition 1 highlights the positional disadvantages of early movers while Proposition 2, from a contest holder's perspective, demonstrates the effect of player positions on the maximum potential contribution they can make to increase the quality of the best solution. These insights will have implications at the contest level, in the investigation of the expected highest quality as well as efficiency in the following sections.

3.2 Expected highest quality

We next derive the expressions for the expected highest quality in both simultaneous and sequential contests in our setting, and examine their comparative statics with respect to the total number of

[8] This latter point could be verified by taking the second derivative of player i's expected payoff $q_i^{1-(1-c)^{n-i}} - q_i$ with respect to q_i, and taking the first derivative of the resulted expression with respect to i.

participants n and the distribution parameter c.

3.2.1 Simultaneous contests

The expected highest quality in simultaneous contests have been derived previously by Moldovanu and Sela (2006). Here, we reproduce their result under our model setup.

$$\text{EHQ}^{Sim} = \frac{n(n-1)c^2}{(1+(n-1)c)(2nc-c+1)} \tag{3}$$

Although prior research has shown that the expected highest quality is not monotonic in general (Archak and Sundararajan, 2009; Moldovanu and Sela, 2006), in our case, as stated in Proposition 3, EHQ^{Sim} monotonically increases in both n and c, which is also illustrated in Figure 2, in which both curves marked with "Sim" (for $n = 3$ and $n = 20$ respectively) increase in c, and the one with $n = 20$ is clearly above $n = 3$ for the full range of $c \in (0, 1)$.[9]

Proposition 3. *The expected highest quality in simultaneous contests monotonically increases in both the total number of contestants, n, and the talent level in the participant pool, c.*

Proof. See Appendix C. □

Asymptotically, as n increases to infinity, the expected highest quality converges to $\frac{1}{2}$ (Archak and Sundararajan, 2009; Chawla et al., 2012; Moldovanu and Sela, 2006). And as c increases to 1, the expected highest quality converges to $\frac{n-1}{2n}$. Again, when n is large, $\frac{n-1}{2n}$ converges to $\frac{1}{2}$.

Proposition 3 indicates that, in our particular model setup, as competition increases (due to either an increase in the number of participants or an increase in the talent level of the participant pool), the combined effect of the positive and negative incentive effects, and the parallel path effects (i.e., a higher n or a higher c leads to higher chances of discovering talents) is positive, leading to monotonic increases in the expected highest quality.

3.2.2 Sequential contests

The explicit solution for expected highest quality for the cases of $n = 2, 3,$ and 4 have been derived by Segev and Sela (2014a). In Proposition 4, we generalize their result to n-player cases:

Proposition 4. *The expected highest quality in n-player sequential contests is:*

$$EHQ^{Seq} = \sum_{i=1}^{n}\sum_{j=i+1}^{n} \frac{c\left((1-d_{j-1})^{\frac{1+(1-c)^{n-j+1}-(1-c)^n}{d_{j-1}}} - (1-d_j)^{\frac{1+(1-c)^{n-j+1}-(1-c)^n}{d_j}}\right)}{\left(\prod_{m=1}^{j-1}(1-d_m)^c\right)\left(\frac{1+(1-c)^{n-j+1}-(1-c)^n}{d_i}\right)}$$

Also, EHQ^{Seq} has an upper bound of $1/e$.

[9] We should note that as a limiting case of ours ($c = 1$), Chawla et al. (2012) show that with a uniform distribution, the expected highest quality monotonically increases in n.

Proof. See Appendix D. □

EHQ^{Seq} monotonically increases in n, although such monotonicity does not hold for general ability distributions (Segev and Sela, 2014a). Counterintuitively, EHQ^{Seq} is not monotonic in c (recall that a higher c means more higher ability participants). This result has been shown by Segev and Sela (2014a), who demonstrate that a group of relatively weak participants may generate a higher expected highest quality than a group of strong participants with the same group size. Figure 2 illustrates these comparative statics under our specific model setup, which clearly shows that there exists a $c^* \in (0, 1)$ (for the cases of $n = 3$ and $n = 20$ respectively) that maximizes the expected highest quality.[10] This result is in stark contrast to Proposition 3, which shows that a higher c always leads to a higher expected best quality in simultaneous contests. That is, if the talent level increases in the participant pool, a simultaneous contest will surely benefit from this increase but a sequential contest may not.

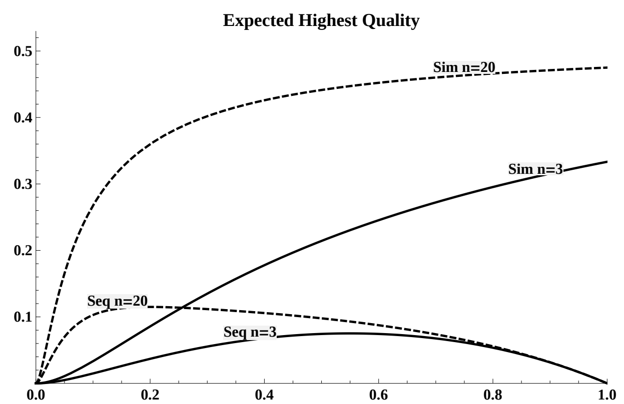


Figure 2: The expected highest quality in simultaneous and sequential contests as c increases from 0 to 1.

3.2.3 Comparing simultaneous and sequential contests

The two popular mechanisms for crowdsourcing, simultaneous and sequential contests, produce different outcomes. Here, in our model setup with n symmetric players, we show that simultaneous contests always generate better highest quality solutions than sequential contests.[11]

Theorem 1. *For any given number of participants, n, and any given talent level $c \in (0, 1)$ in the participant pool, the expected highest quality is higher in a simultaneous contest than in a sequential contest.*

Proof. See Appendix E. □

[10]This can be shown using the extreme value theorem.

[11]Segev and Sela (2014b) discuss a numerical example with two players that shows that a simultaneous contest produces significantly higher expected highest quality than a sequential contest.

This result has been foreshadowed by our previous analysis on the equilibrium strategies. As Proposition 2 indicates, in sequential contests, the order of entry, especially for those with high ability, plays an important role. Depending on their ability levels, entering at different stages can have great effects on their equilibrium effort. For example, if a high ability contestant enters a contest early, her capacity is maximized as she has to work hard to win, in anticipation of the competition ahead. On the other hand, if a high ability contestant enters late, she may be able to exert less effort and win the contest with a bigger surplus. This effect echoes what has been observed by Segev and Sela (2014a), who show that a contest holder who can determine the order of entry for all participants (in their model the participants' abilities are asymmetric and the contest holder can determine the order of entry) would never want to put a strong participant in the last stage.

Theorem 1 does not imply, however, that simultaneous contests always generate better best solutions than sequential contests *ex post*. This can be seen from the following numerical example.

Example 1. *Given* $c = 0.25$*,* $n = 3$*,* $a_1 = 0.004$*,* $a_2 = 0.009$*, and* $a_3 = 0.001$*, in the simultaneous game, player 2 produces the highest effort* $q_2^{Sim} = 0.00028$*. In the sequential game,* $q_1^{Seq} = 0.000013$*,* $q_2^{Seq} = 0.00029$*, the highest effort is higher than the highest effort in the simultaneous contest.*

Without deriving the explicit expression for the expected total effort in sequential contests, [12] we can borrow Myerson's (1981) analysis to gain some insights into the relative performance between the two mechanisms in inducing higher expected total effort. The analysis by Myerson (1981) implies that when a regularity condition is satisfied (i.e., contestant i's virtual value, $a - \frac{1-F_i(a)}{f_i(a)}$, is an increasing function of a), the optimal mechanism should always allocate the prize to the contestant with the highest ability. [13] Since the allocation rule of sequential contests is influenced by contestants' positions, it does not always allocate the prize to the most capable contestant. Therefore, sequential contests are inferior to simultaneous contests in inducing total efforts. Now, due to the particular distribution function used in our study, $F(a) = a^c$, the regularity condition is violated. Therefore, Myerson's (1981) result does not apply directly to our case. However, it can be shown that asymptotically (n sufficiently large), simultaneous contests should induce higher expected total effort than sequential contests, as formally stated in the following Proposition:

Proposition 5. *Given the homogeneous distribution function* $F(a) = a^c, a \in (0, 1)$ *among all contestants, for any* $c \in (0, 1)$*, there exists an* N*, such that for any* $n > N$*, simultaneous contests generate higher expected total effort than sequential contests.*

Proof. See Appendix F. □

[12] The expected total effort in sequential contests is intractable.

[13] Strictly speaking, Myerson's (1981) analysis requires that the contestant with the highest virtual value ($a_i - \frac{1-F_i(a_i)}{f_i(a_i)}$) wins. In our setting with identical ability distributions among players ($F_i(a) = F(a)$), the optimal mechanism should then just make the contestant with highest ability win.

3.3 Efficiency

To evaluate the efficiency of a contest, our first concern is allocative efficiency, e.g., how often a contest produces an efficient allocation. In Segev and Sela's (2014b) study, the *proportion of efficient allocations* is used to measure allocative efficiency. Another measure, suggested by Plott and Smith (1978), is the ratio of the winning contestant's ability to the highest ability among all contestants. In some literature this measure is simply called "efficiency" (e.g., Noussair and Silver, 2006). Here, we call it *value efficiency* to differentiate it from the general term, efficiency, that we use to refer to multiple measures in our study.

A simultaneous contest always produces the most efficient outcome: As all contestants are symmetric and the equilibrium effort function monotonically increases in ability, the player with the highest ability always wins and full efficiency is always achieved (Archak and Sundararajan, 2009). Therefore, simultaneous contests score one on both allocative efficiency measures.

Sequential contests, however, are not always efficient, as late movers have positional advantages (see Proposition 1). As a numerical example, in a three-player contest with $c = 0.5$, if $a_1 = 0.2$ and $a_2 = 0.05$, player 2's BNE effort will be 0.0006, which is higher than player 1's BNE effort, 0.0005. Obviously, the lower allocative efficiency in sequential contests is driven by the randomness in the order of entry. From a contest holder's perspective, the imperfect allocative efficiency in sequential contests suggests that in some cases, the same set of contestants could have produced a better outcome had the contestants followed a different order of entry. From a contestant's perspective, simultaneous contests might seem more "fair" than sequential contests, since the most talented contestants always win in simultaneous contests. In sequential contests, however, low ability contestants might have better luck in winning than in simultaneous contests.

Unfortunately, for sequential contests, the analytical solutions for both allocative efficiency measures are intractable for the general n-player case.[14] Here, for the special cases of $c = 0.25$, $n = 2$ and $n = 3$ (the parameter values used in our experimental study), we produce their numerical solutions (shown in Table 2). At least for these special cases, we see that both allocative efficiency measures are substantially lower in sequential contests. Additionally, both measures decrease in n, making sequential contests appear worse in comparison to simultaneous ones.

In addition to allocative efficiency, another important concern, especially from a social planners' perspective, is the amount of effort wasted in a contest. Recall that in contrast to an all-pay auction, in which all losers' bids are collected by the auctioneer as part of the revenue, in a contest, all losers' effort is wasted. From a social planner's perspective, the less waste, the better. To quantify the amount of effort wasted in a contest, Chawla et al. (2012) define *utilization ratio* as the ratio of expected total effort by all participants (ETQ) to the expected highest effort (EHQ), i.e., $\frac{ETQ}{EHQ}$. Essentially, it measures how much effort has to be made by all participants, for every unit of gain by the contest holder from the highest quality solution. The higher the utilization ratio, the more wasteful a contest is.

[14] Segev and Sela (2014b) derive the proportion of efficient allocations for the two-player case, and demonstrate that it is lower in sequential contests than in simultaneous contests.

Table 1: Experimental Design

Contest Format	Contest Size	Treatment Name	Number of Subjects/Session	Number of Sessions	Total Number of Subjects
Sequential	3	Seq-G3	12	4	48
	2	Seq-G2	12	4	48
Simultaneous	3	Sim-G3	12	4	48
	2	Sim-G2	12,10,8	4	42

Intuitively, one might expect sequential contests to be less wasteful, thereby having a lower utilization ratio, since participants can view prior submissions and adjust their own effort accordingly. However, this is not necessarily true, as demonstrated by Proposition 6 using a special case of $n = 2$. Even though the absolute amount of total effort might be lower in sequential contests (as shown in Table 2 for the case of $c = 0.25$), the relative amount, weighted against the expected highest effort, can still be higher.

Proposition 6. *When there are two players, the utilization ratio is always higher in sequential contests than in simultaneous contests, for any talent level $c \in (0, 1)$.*

Proof. See Appendix G. □

4 Experimental Design

To test the theoretical predictions, especially to examine the effects of contest mechanism and contest size, we implemented a between-subject 2×2 factorial design, as shown in Table 1. We expected individual behaviors to vary with contest mechanism (simultaneous versus sequential) as well as contest size (the number of participants).

Every treatment had four independent sessions, each with 12 subjects.[15] At the beginning of each round subjects were randomly assigned into groups of two or three, depending on the treatment. As our theoretical model was based on one-shot games, this random re-matching protocol was used to minimize repeated-game effects in the experiment. In all treatments, the value of the reward was 100 tokens and every subject was given an ability factor, a_i ($a_i \in (0, 1]$), which was randomly drawn from $F(a) = a_i^{0.25}$. They were informed that their ability factors were private knowledge. To prevent potential bankruptcies, we gave every subject 120 tokens as an endowment at the beginning of each round and they could choose an effort level from 0 to 120. [16] Finally, to capture any learning effect, each subjects played eighteen rounds after two practice rounds.

[15]Two simultaneous contest sessions with two players (Sim-G2) did not have enough subjects. Therefore, we had to run a session with 10 subjects and another with 8 subjects. However, the observed behaviors in these two sessions are not statistically significantly different from other sessions, so we treated them the same as other sessions of the same treatment.

[16]Four decimal points were kept for the display of ability factors, i.e., a_i, and for subjects' input of their effort levels, i.e., q_i.

In simultaneous contest treatments, every participant made an effort independently and simultaneously. The sequential contests were implemented with games in multiple stages (2 stages for G2 and 3 stages for G3). In each stage a different participant had an exclusive opportunity to submit an effort level (including 0), after observing all previous participants' effort levels. Before the first stage started, the order of entry was randomly assigned by the computer and was announced to all participants in each group. Additionally, in both simultaneous and sequential treatments, before everyone chose an effort level, we elicited their beliefs of their winning probabilities. To incentivize the subjects to accurately report their beliefs, a quadratic scoring rule (Nyarko and Schotter, 2002) was implemented which paid a maximum amount of two tokens for making accurate predictions. A sample of the instructions is included in Appendix H. After the experiment, we gave each participant a post-experiment survey which collected demographic and personality trait information, as well as data on their risk- and loss aversion. The post-experiment questionnaire is included in Appendix I.

Based on our experimental parameter values, i.e., $c = 0.25$ and $n = 2, 3$, we computed all the theoretical values of interest and report them in Table 2. The table includes the equilibrium effort for each player, their winning probability, the expected highest effort, the average effort, the total effort, the expected proportion of efficient allocations, the value efficiency as well as the utilization ratio.

In total, we conducted 16 independent computerized sessions at the Los Angeles Behavioral Economics Laboratory at the University of Southern California from January 2012 to February 2013, utilizing a total of 186 subjects. Our subjects were students recruited by email from a subject pool for economics experiments. We used z-Tree (Fischbacher, 2007) to program our experiments. Each session lasted approximately one and a half hours. The exchange rate was set to 110 tokens per $1. The average amount earned by our participants was $29, including a $5 show-up fee.[17] Our experimental data are available from the authors upon request.

5 Experimental Results

We start with results aggregated at the session level and report the treatment effects with regard to both submission quality and efficiency. A detailed individual-level analysis follows to examine dynamics in participants' behaviors.

5.1 Submission quality

Recall our theoretical result that simultaneous contests produce better highest quality solutions than sequential contests (Theorem 1), which leads to Hypothesis 1:

[17]Seven students went bankrupt during the experiment and we decided to pay them a ten dollar flat fee to compensate them for the time, although such a payment was not pre-announced.

Table 2: Theory Predictions with $c = 0.25$

N	Game	Expected Quality			Winning Probability		
		Player 1	Player 2	Player 3	Player 1	Player 2	Player 3
2	Seq	2.487	1.193	NA	27%	73%	NA
3	Seq	2.836	2.835	2.069	13%	25%	63%
2	Sim	3.333	3.333	NA	50%	50%	NA
3	Sim	4.762	4.762	4.762	33%	33%	33%
N	Game	Expected Quality			% of Efficient	Value	Utilization
		Highest	Average	Total	Allocations	Efficiency	Ratio
2	Seq	2.487	1.840	3.68	77%	86%	1.480
3	Seq	4.684	2.580	7.74	65%	81%	1.652
2	Sim	5.714	3.333	6.666	100%	100%	1.167
3	Sim	11.111	4.762	14.286	100%	100%	1.286

Hypothesis 1 (Submission quality comparison)**.** *For a given number of contestants and a given talent level in the participant pool, the expected highest quality is higher in simultaneous than in sequential contests.*

Figure 3 plots the mean of the highest quality under sequential and simultaneous contests in each round (The left panel is for contests with two players and the right one is for contests with three players). Clearly, in contests with two players, simultaneous contests outperform sequential contests throughout all rounds, whereas the pattern in contests with three players is less clear at the beginning. However, toward the second half of the session (round 9 onwards), it becomes clear that simultaneous contests produce better highest quality solutions than sequential contests. We summarize this result below:

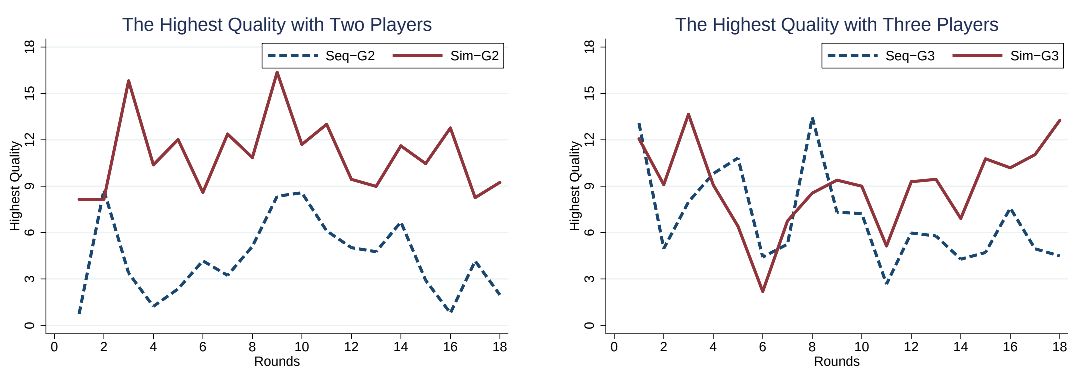


Figure 3: The Highest Quality: Sequential vs. Simultaneous Contests

Result 1 (Submission quality comparison)**.** *The highest quality is higher in simultaneous contests than in sequential contests.*

Support. *Table 3 shows the means of the highest quality in different treatments for all rounds as well as for the first (Rounds 1-9) and second half (Rounds 10-18) of the session separately. On average, the highest quality is marginally significantly higher in Sim-G2 than in Seq-G2 (All Rounds: 11.01 vs. 4.35,* $p = 0.083$*, one-sided permutation tests). Consistently, the highest quality in Sim-G3 is higher than that in Seq-G3. In particular, the comparison becomes statistically significant in the second half of the experiment (Rounds 10-18: 9.45 vs. 5.30,* $p = 0.030$*, one-sided permutation tests).*

Table 3: Treatment Effects on the Highest Quality

All Rounds	Seq	Sim	Seq vs. Sim
n=2	4.35	11.01	$p = 0.083$
n=3	6.93	9.01	$p = 0.172$
n=2 vs. n=3	$p = 0.097$	$p = 0.436$	
Rounds 1-9	Seq	Sim	Seq vs. Sim
n=2	4.14	11.41	$p = 0.055$
n=3	8.57	8.58	$p = 0.501$
n=2 vs. n=3	$p = 0.086$	$p = 0.378$	
Rounds 10-18	Seq	Sim	Seq vs. Sim
n=2	4.55	10.61	$p = 0.087$
n=3	5.3	9.45	$p = 0.030$
n=2 vs. n=3	$p = 0.299$	$p = 0.465$	

Note: P-values from one-sided permutation tests are reported for all comparisons in this table.

By Result 1, we reject the null in favor of Hypothesis 1. The difference between the two mechanisms' performance is quite large. For example, in two-player contests, the mean highest quality in simultaneous contests is 253% higher than that in sequential contests (11.01 vs. 4.35, All Rounds).

To benchmark our experimental contests' performance against their BNE predictions, treating each session as an independent observation, we report one-sided p-values from one-sample signed rank tests. In simultaneous contests, the aggregate highest quality does not appear different from BNE predictions ($p > 0.1$, one-sided one-sample signed rank tests) but in sequential contests, they are marginally significantly higher than BNE predictions (Seq-G2: 4.35 vs. 2.49, $p = 0.07$; Seq-G3: 6.93 vs. 4.68, $p = 0.07$, one-sided one-sample signed rank tests). Nevertheless, even though sequential contests perform better than predicted by theory, they still produce lower quality best solutions than simultaneous contests.

Consistent with Proposition 2, we find that the sequence of entry does affect the submission quality in sequential contests. Specifically, in the Seq-G2 treatment, the average highest submission quality is higher when the higher-ability contestant enters early instead of late (7.19 vs. 1.54, $p = 0.034$, one-sided signed rank tests). Consistently, in Seq-G3 treatment, a contest in which the highest-ability contestant enters first, followed by a medium-ability player and then the lowestability one, generates significantly higher best quality than all other entry sequences (11.13 vs. 6.35, $p = 0.034$, one-sided signed rank tests).

In addition, consistent with the theoretical values computed for our experimental setting (see Table 2), we find that simultaneous contests also produce higher average quality solutions than sequential contests. Although the comparison is not statistically significant when all rounds are considered (Sim-G2 vs. Seq-G2: 5.96 vs. 2.91, $p = 0.128$; Sim-G3 vs. Seq-G3: 3.86 vs. 3.21, $p = 0.245$, one-sided permutation tests), it becomes significant when only the second half of the experiment is considered in contests with three players (Sim-G3 vs. Seq-G3: 4.21 vs. 3.93, $p = 0.014$, one-sided permutation tests).[18] Taken together, these results suggest that simultaneous contests are more effective than sequential contests in inducing higher best quality and average quality from their contestants.

By and large, these results are consistent with prior empirical findings comparing simultaneous and sequential contests. All of the prior studies report that simultaneous contests lead to higher average effort (Boudreau and Lakhani, 2014; Liu, 2013) or quality (Wooten and Ulrich, 2013). Particularly noteworthy in Wooten and Ulrich's (2013) study are the differential effects of contest mechanism on participants' submission quality: only high ability contestants do better in simultaneous contests and low ability ones do worse. This observation, we argue, could suggest higher quality *best* solutions in simultaneous contests (consistent with our results), if we assume that higher skills do lead to better quality solutions.

Next, we examine the size effect on submission quality. Following Proposition 3 and results by Segev and Sela (2014a), we propose Hypothesis 2:

Hypothesis 2 (Size effect on submission quality)**.** *In both sequential and simultaneous contests, the expected highest quality increases in the number of contestants.*

Again, results are reported in Table 3. On average, the highest quality is marginally significantly higher in the Seq-G3 treatment than in the Seq-G2 treatment (All Rounds: 6.93 vs. 4.35, $p = 0.097$, one-sided permutation tests) but the comparison becomes statistically nonsignificant in the second half of the experiment (Rounds 10-18: 5.30 vs. 4.55, $p = 0.299$, one-sided permutation tests). Furthermore, no statistically significant difference is observed between Sim-G2 and Sim-G3 treatments ($p > 0.1$, for All Rounds, Rounds 1-9 and Rounds 10-18) either. Therefore, we cannot reject the null in favor of Hypothesis 2 in our experiment.

Additionally, even though the theoretical values computed in Table 2 suggest that both simultaneous and sequential contests should produce higher average quality when n increases, this prediction is not supported by our data. Although the average quality is marginally significantly lower in Seq-G2 than in Seq-G3 (All Rounds: 2.91 vs. 3.21, $p = 0.082$, one-sided permutation tests), the difference is not significant in the second half of the experiment (Rounds 10-18: 3.12 vs. 2.62, $p = 0.199$, one-sided permutation tests). The comparison of the average quality between the two simultaneous contests is not statistically significant either ($p > 0.1$, one-sided permutation

[18]The comparison remains nonsignificant for the contests with two players (Sim-G2 vs. Seq-G2: 5.80 vs. 3.12, $p = 0.131$, one-sided permutation tests).

tests). Taken together, we suspect that adding one more player to contests cannot significantly alter contestants' effort.

5.2 Efficiency

The analysis on efficiency in Section 3.3 suggests that simultaneous contests are more efficient than sequential contests, leading to Hypothesis 3:

Hypothesis 3 (Efficiency comparison)**.** *For a given number of contestants and a given talent level in the participant pool, simultaneous contests achieve higher efficiency than sequential contests.*

Table 4: Treatment Effects on the Proportion of Efficient Allocations

All Rounds	Seq	Sim	Seq vs. Sim
n=2	0.75	0.78	$p = 0.209$
n=3	0.56	0.74	$p = 0.014$
n=2 vs. n=3	$p = 0.014$	$p = 0.131$	
Rounds 1-9	**Seq**	**Sim**	**Seq vs. Sim**
n=2	0.74	0.78	$p = 0.139$
n=3	0.57	0.73	$p = 0.055$
n=2 vs. n=3	$p = 0.028$	$p = 0.224$	
Rounds 10-18	**Seq**	**Sim**	**Seq vs. Sim**
n=2	0.75	0.77	$p = 0.363$
n=3	0.55	0.75	$p = 0.029$
n=2 vs. n=3	$p = 0.029$	$p = 0.293$	

Note: P-values from one-sided permutation tests are reported for all comparisons in this table.

Tables 4, 5 and 6 present the proportion of efficient allocations, value efficiency, and utilization ratio in different treatments for all rounds as well as for the two halves of the sessions separately. Comparing the efficiency measures across treatments, we find that, consistent with theoretical predictions, simultaneous contests produce more efficient allocations as well as higher value efficiency than sequential contests. The utilization ratio (waste level) is also lower in simultaneous contests. We summarize the results below:

Result 2 (Efficiency comparison)**.** *Across all three efficiency measurements, simultaneous contests are superior to sequential contests.*

Support. *All test results are reported in Tables 4, 5 and 6. The proportion of efficient allocations is higher in simultaneous contests than in sequential contests. In particular, the comparison for the three-player case produces statistically significant results (Sim-G3 vs. Seq-G3: All Rounds: 0.74 vs. 0.56,* $p = 0.014$*; Rounds 1-9: 0.73 vs. 0.57,* $p = 0.055$*; Rounds 10-18: 0.75 vs. 0.55,* $p = 0.029$*, one-sided permutation tests). Similarly, in three-player conditions, value efficiency is also significantly higher in simultaneous contests (Sim-G3 vs. Seq-G3: All Rounds: 0.86 vs.*

0.7, $p = 0.014$; Rounds 1-9: 0.84 vs. 0.7, $p = 0.04$; Rounds 10-18: 0.89 vs. 0.7, $p = 0.014$, one-sided permutation tests). Additionally, the utilization ratio is lower in simultaneous contests than in sequential contests. The comparison in the two-player case is statistically significant in all specifications (Sim-G2 vs. Seq-G2: All Rounds: 1.11 vs. 1.35, $p = 0.014$; Rounds 1-9: 1.08 vs. 1.31, $p = 0.014$; Rounds 10-18: 1.13 vs. 1.38, $p = 0.014$, one-sided permutation tests), and in three-player cases, the comparison is statistically significant when all rounds are considered (Sim-G3 vs. Seq-G3: All Rounds: 1.30 vs. 1.40, $p = 0.041$, one-sided permutation tests).

Table 5: Treatment Effects on Value Efficiency

All Rounds	Seq	Sim	Seq vs. Sim
n=2	0.84	0.86	$p = 0.251$
n=3	0.70	0.86	$p = 0.014$
n=2 vs. n=3	$p = 0.014$	$p = 0.412$	
Rounds 1-9	**Seq**	**Sim**	**Seq vs. Sim**
n=2	0.84	0.85	$p = 0.348$
n=3	0.70	0.84	$p = 0.040$
n=2 vs. n=3	$p = 0.014$	$p = 0.298$	
Rounds 10-18	**Seq**	**Sim**	**Seq vs. Sim**
n=2	0.83	0.86	$p = 0.291$
n=3	0.70	0.89	$p = 0.014$
n=2 vs. n=3	$p = 0.029$	$p = 0.112$	

Note: P-values from one-sided permutation tests are reported for all comparisons in this table.

Due to Result 2, we reject the null hypothesis in favor of Hypothesis 3 and conclude that simultaneous contests are superior to sequential contests by not only creating a higher probability for high ability contestants to win, but also by generating a relatively lower amount of wasted effort from losers.

To benchmark the efficiency measures with their theoretical predictions shown in Table 2, we report results using one-sample signed rank tests. Overall, simultaneous contests' allocative efficiency levels are lower than expected, e.g., achieving a proportion of efficient allocations lower than 100% (Sim-G2: 78%, $p = 0.034$; Sim-G3: 74%, $p = 0.034$, one-sided tests). Seq-G2 contests' allocative efficiency levels are very close to their theoretical predictions, e.g., reaching a proportion of efficient allocations at 75% (vs. 77%, $p = 0.13$, one-sided test). The Seq-G3 contests, however, are less efficient than predicted, e.g., achieving a proportion of efficient allocations of 56% (vs. 65%, $p = 0.035$, one-sided test). In terms of utilization ratio, simultaneous contests perform well by achieving a mean of 1.11 in the Sim-G2 treatment (vs. 1.17, $p = 0.035$, one-sided test) and 1.30 in the Sim-G3 treatment (vs. 1.29, $p = 0.35$, one-sided test). Sequential contests also reach utilization ratios lower than predicted (Seq-G2: 1.35 vs. 1.48, $p = 0.035$; Seq-G3: 1.40 vs. 1.65, $p = 0.035$, one-sided tests). Taken together, all treatments' allocative efficiency levels are lower than predicted, except the Seq-G2 treatment and all their utilization ratios are not higher than predicted, thus not generating more wasted effort than predicted by theory.

Table 6: Treatment Effects on Utilization Ratio

All Rounds	Seq	Sim	Seq vs. Sim
n=2	1.35	1.11	$p = 0.014$
n=3	1.40	1.30	$p = 0.041$
n=2 vs. n=3	$p = 0.229$	$p = 0.014$	
Rounds 1-9	**Seq**	**Sim**	**Seq vs. Sim**
n=2	1.31	1.08	$p = 0.014$
n=3	1.26	1.23	$p = 0.362$
n=2 vs. n=3	$p = 0.272$	$p = 0.014$	
Rounds 10-18	**Seq**	**Sim**	**Seq vs. Sim**
n=2	1.38	1.13	$p = 0.014$
n=3	1.53	1.35	$p = 0.112$
n=2 vs. n=3	$p = 0.200$	$p = 0.014$	

Note: P-values from one-sided permutation tests are reported for all comparisons in this table.

Last, we examine the size effect on efficiency. We do not expect any size effect on allocative efficiency in simultaneous contests, since simultaneous contests are always efficient regardless of contest size. In sequential contests, as shown in Table 2, we should expect efficiency to decrease in n. As for the utilization ratio, we expect it to increase (become less efficient) as n increases in both simultaneous and sequential contests, as suggested by the predicted values in Table 2. Summarizing, we have the next hypothesis:

Hypothesis 4 (Size effect on efficiency)**.** *In sequential contests, all three efficiency measures worsen as the number of contestants increases. In simultaneous contests, the utilization ratio worsens as the number of contestants increases.*

Highly consistent with theoretical predictions, we find that in sequential contests both measures of allocative efficiency decrease as the number of contestants increases, and in simultaneous contests no size effect is observed. In addition, for simultaneous contests, we find that the utilization ratio (waste level) increases with the number of players. We summarize these results below:

Result 3 (Size effect on efficiency)**.** *Both the proportion of efficient allocations and value efficiency in sequential contests decrease with the number of contestants. The amount of wasted effort increases with the number of contestants in simultaneous contests.*

Support. *In Table 4, the proportion of efficient allocations in sequential contests with two players is statistically significantly higher than those with three players (All Rounds: 0.75 vs. 0.56,* $p = 0.014$*; Rounds 1-9: 0.74 vs. 0.57,* $p = 0.028$*; Rounds 10-18: 0.75 vs. 0.55,* $p = 0.029$*, one-sided permutation tests). Consistently, in Table 5, the value efficiency in sequential contests with two players is statistically significantly higher than those with three players (All Rounds: 0.84 vs. 0.7,* $p = 0.014$*; Rounds 1-9: 0.84 vs. 0.7,* $p = 0.014$*; Rounds 10-18: 0.83 vs. 0.7,* $p = 0.029$*, one-sided* *permutation tests). In addition, the utilization ratio in simultaneous contests with three players is significantly higher than those with two players (All Rounds: 1.30 vs. 1.11, $p = 0.014$; Rounds 1-9: 1.23 vs. 1.08, $p = 0.014$; Rounds 10-18: 1.35 vs. 1.13, $p = 0.014$, one-sided permutation tests).*

By Result 3, for simultaneous contests, we reject the null in favor of Hypothesis 4. For sequential contests, we observe a significant size effect on the two allocative efficiencies, but not for the utilization ratio. These results suggest that contests with two players are at least as good as those with three players. In particular, in sequential contests, adding one more contestant can reduce their efficiency significantly, e.g., the proportion of efficient allocations dropping from 0.75 to 0.55 in the second half of the sessions, although such differences are not observed for simultaneous contests. Such differential size effects in these two mechanisms highlight the impact of participant asymmetry in sequential contests, such that efficiency in sequential contests is highly sensitive to contest size. Furthermore, with regard to utilization ratio, we observe that as n increases, simultaneous contests become more wasteful. It remains to be studied whether the utilization ratio of sequential contests can become lower than simultaneous contests' when n becomes large.

5.3 Individual level analysis

In this section, we start by examining individuals' winning probabilities. Consistent with Proposition 1, we find that late entrants in sequential contests have much higher winning probabilities than early entrants. Specifically, in the Seq-G2 treatment, the first player wins 25% of the time and the last player 75% (close to the theory predicted 27% vs. 73%), and the first player's chance of winning is significantly lower than the last player's ($p < 0.01$, one-sided test of proportions). The comparisons in Seq-G3 also yield statistically significant results (First vs. Last: 14% vs. 65%, $p < 0.01$; Middle vs. Last: 22% vs. 65%, $p < 0.01$, one-sided test of proportions).[19] These observed winning probabilities, i.e., 14%, 22%, and 65%, are again similar to their predicted values: 13%, 25%, and 63% (shown in Table 2).

Our analysis of individuals' strategies starts by checking whether the last players in sequential contests best respond, since their strategies are the most straightforward — they either fold or call, depending on their abilities and the current highest effort. We find that the majority of last players (77% in Seq-G2 and 65% in Seq-G3) in sequential contests best respond. There is also a strong indication of learning: in the second half of the experiment, the percentage of best responses increases to 83% in Seq-G2 and to 74% in Seq-G3.[20]

Next, we examine all other players' strategies, except the middle players' in the Seq-G3 con-

[19]The comparison of winning probability between the middle player and the last player is also statistically significant (14% vs. 22%, $p = 0.011$, one-sided test of proportions).

[20]Because z-Tree requires a maximum decimal point for any numerical input, subjects' efforts were restricted to four decimal points in our experiment. Therefore, we use 0.0001 as the increment of best responses, i.e., when a player plays the call strategy, she should simply make an effort that equals the existing highest effort, plus 0.0001. To account for subjects' imprecision in entering their efforts, we checked our results using 0.001, 0.01 and 0.1, and found no significant changes to our results.

Table 7: The Deviation from BNE Predictions: OLS Regressions

	Simultaneous		Sequential	
	N=2	N=3	N=2	N=3
	(1)	(2)	(3)	(4)
Ability	-4.05	-10.50**	2.75	-0.17
	(1.99)	(2.81)	(3.29)	(3.38)
Period	0.02	-0.02	0.01	-0.10
	(0.06)	(0.06)	(0.09)	(0.15)
Constant	3.07	1.41*	1.02	1.65
	(0.58)	(0.58)	(1.01)	(1.76)

Notes: The standard errors are clustered at the session level.

** $p < 0.05$, * $p < 0.1$

dition (which will be discussed later), since their choices will have to depend on their preceding players'. Table 7 presents the OLS regression results, predicting the deviation of individuals' observed effort from their BNE predictions (i.e., $\delta =$ actual effort $-$ predicted effort) for all players in simultaneous contests and the first players in sequential contests. The independent variables include the ability factor and the period variable to control for learning effects.[21] The results show that except in the Sim-G3 treatment, no estimated coefficients are significantly different from zero. In the Sim-G3 treatment, both coefficients on the constant term and the ability factor are statistically significantly different from zero ($p < 0.1$ for the former and $p < 0.05$ for the latter). The signs indicate that for low ability participants, there is over-investment of effort (the coefficient on the constant term, 1.41, is positive) but the extent of over-investment decreases in the ability factor (the coefficient on the ability factor, -10.50, is negative), eventually leading to under-investment.

To get a sense of the absolute amount of over- or under-investment by participants across all treatments, we compute the mean deviation, $\bar{\delta}$, by the four quantiles of the ability distribution and find that only high ability players in the Sim-G3 condition exert significantly lower effort than BNE predictions. That is, in the highest quantile of the ability distribution, i.e., $a > 0.32$, the average deviation is -5.05 and the difference is significantly lower than zero ($p = 0.034$, one-sided one-sample signed rank tests).[22]

Altogether, these results reveal that with two contestants, the observed effort levels in simultaneous contests match their BNE predictions, in line with findings from previous experiments using similar experimental protocols (Grosskopf et al., 2010; Potters et al., 1998). In contrast, with three contestants, the effort levels of high ability contestants in simultaneous contests are significantly lower than BNE predictions. This contradicts previous findings (Muller and Schotter, 2010;

[21]We also ran probit regressions where the dependent variable is the likelihood of overbidding, and the results are qualitatively consistent.

[22]Following Muller and Schotter (2010), we used a switching regression model to examine whether the individuals' effort function was continuous. The switching regression model fit the data significantly better than BNE predictions based on the sum of squared deviations (SSD) measure (Sim-G2: 21.37 vs. 549.83, $p < 0.01$; Sim-G3: 25.04 vs. 695.77, $p < 0.01$, one-sided signed-rank tests). Nevertheless, this switching regression model could not explain why the effort levels of high ability contestants in Sim-G3 were lower than BNE predictions.

Noussair and Silver, 2006) where high ability contestants (from a uniform distribution) over-invest their effort. Muller and Schotter (2010) rationalize their data by assuming loss averse agents [23] and Noussair and Silver (2006) explain the bidding pattern by Fibich et al.'s (2006) model of risk aversion. However, neither of these models can explain the seemingly counterintuitive result here. Below, we discuss why under-investment is observed for high ability contestants in the Sim-G3 treatment, but not in the Sim-G2 treatment and not for high ability first players in sequential contests.

We conjecture that the under-investment in effort by high ability contestants in Sim-G3 contests is due to overplacement, a specific type of overconfidence (Moore and Healy, 2008) — people tend to overestimate their chances of success in comparison to their average peers — in particular driven by our choice of power distribution for the contestants' ability factors. Although the power distribution is a good approximation for the empirical distribution of participants' abilities in crowdsourcing markets (Araujo, 2013; Yang et al., 2008), it is relatively hard for experimental participants to work with, compared to the uniform distribution function often used in previous experiments (e.g., Muller and Schotter, 2010; Noussair and Silver, 2006).[24] In particular, the power distribution function features a larger proportion of low ability contestants, which makes a draw of relatively high ability a rare event. We suspect that it could easily lead to a contestant's overestimation of her winning probability whenever a high ability factor is drawn for her.

Our first piece of evidence to support the overconfidence effect comes from the belief data (recall that in the experiment we elicited subjects' beliefs of their winning probabilities before letting them make their effort choices). Using this data, we can verify whether the subjects had misbeliefs about their winning probabilities. Indeed, in Sim-G3, subjects believed they were 7% more likely to win than predicted by BNE and this difference is marginally significant ($p = 0.072$, one-sided signed rank tests). In contrast, in Sim-G2, no statistically significant overestimate of winning probabilities was observed (-1%, $p = 0.358$, one-sided signed rank tests). This is understandable, since it is more difficult to compute one's winning probability in a three-player contest than in a two-player one. Additionally, overplacement is more likely to occur if one judges herself against the average of a group of others, compared to against another individual (Alicke et al., 2004), which explains why overconfidence does not occur in the Sim-G2 treatment but only in the Sim-G3 treatment.

To further quantify this overconfidence, we define individual players' misperceived winning probabilities as follows:

1. In Sim-G2, it is $\widetilde{P}(a_i) = a_i^{\widetilde{c}}$ for $i \in \{1, 2\}$;
2. In Sim-G3, it is $\widetilde{P}(a_i) = a_i^{2\widetilde{c}}$ for $i \in \{1, 2, 3\}$;
3. In Seq-G2, it is $\widetilde{P}(a_1) = (a_1 * \widetilde{c})^{\frac{\widetilde{c}}{1-\widetilde{c}}}$;

[23] Under the assumption of loss-aversion, Mermer (2013) analytically shows that high ability contestants over-exert effort while low ability contestants under-exert effort.

[24] The power distribution used in our experiment has been described by its quantiles in the experimental instructions. See Appendix H for details.

4. In Seq-G3, it is $\widetilde{P}(a_1) = (a_1 * (2\widetilde{c} - \widetilde{c}^2))^{\frac{2\widetilde{c}-\widetilde{c}^2}{1-\widetilde{c}^2}}$,

where $\widetilde{c}$ represents the corresponding talent parameter c in the misperceived winning probability. Apparently, the lower the $\widetilde{c}$, the stronger the overconfidence, i.e., the extent to which the probability of winning is over-estimated. Using non-linear regression models, we obtain the point estimations of $\widetilde{c}$ based on the belief data (see results reported in Table 8). We find that the best fitting $\tilde{c}$ in the Sim-G3 treatment is 0.18, which is significantly smaller than 0.25 ($p = 0.03$), while in the Sim-G2 treatment, the estimated $\tilde{c}$ is exactly 0.25. Therefore, there exists significant overconfidence among participants in the Sim-G3 condition.

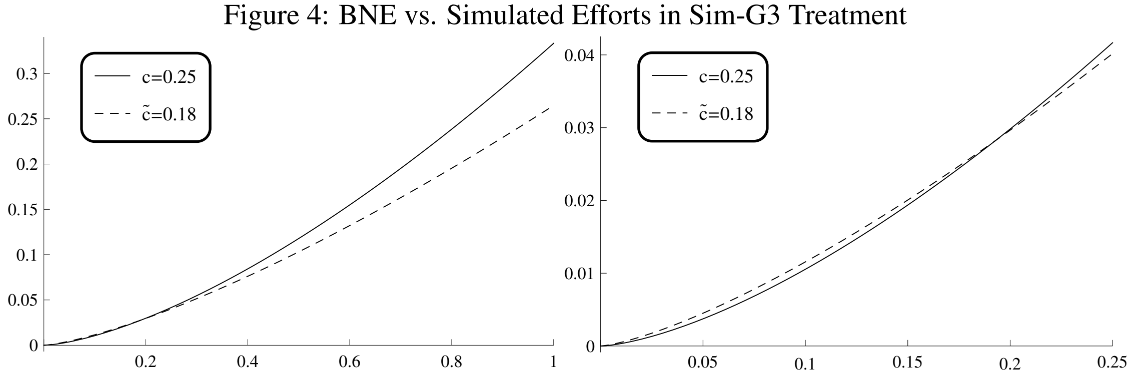
Table 8: The Estimated $\tilde{c}$ from Belief Data

	Simultaneous		Sequential	
	N=2	N=3	N=2	N=3
	(1)	(2)	(3)	(4)
$\tilde{c}$	0.25	0.18	0.21	0.15
Std.Err.	0.03	0.02	0.01	0.01
P-value	0.5	0.03	0.03	0.00
R^2	0.79	0.79	0.66	0.60

Note: P-values are reported from one-sided t-tests with the null hypothesis $\tilde{c} = 0.25$.

Furthermore, we use this best fitting $\tilde{c}$ to simulate participants' corresponding effort choices. For simplicity, we use the BNE effort function to generate the efforts, while replacing the actual c with the estimated $\tilde{c}$. Both the simulated efforts and BNE predicted efforts are plotted in Figure 4. The solid line represents BNE efforts calculated with $c = 0.25$ and the dotted line represents efforts simulated with $\tilde{c} = 0.18$ (The left panel is for the full range of $a \in [0, 1]$ and the right panel shows the same curves but zooms in on the low ability range, i.e., $a \in [0, 0.25]$). We observe that the simulated efforts are higher than BNE predictions in the low ability range but become lower than BNE in the high ability range. This pattern is consistent with the previously described pattern of actual observed efforts, indicating that overconfidence can be a plausible explanation for high ability contestants' under-investment behavior in the Sim-G3 treatment.

Figure 4: BNE vs. Simulated Efforts in Sim-G3 Treatment



Furthermore, to assess model fit, we follow Muller and Schotter (2010) by computing the sum of squared deviations (SSD) of the BNE model (BNE) and the overconfidence model based on belief data (BLF). That is, for each individual i and across all t rounds, $SSD_i^{\text{BLF}} = \sum_{t=1}^{18}((q_i^t)^{BLF} - (q_i^t)^{obs})^2$ and $SSD_i^{\text{BNE}} = \sum_{t=1}^{18}((q_i^t)^{BNE} - (q_i^t)^{obs})^2$, where $(q_i^t)^{obs}$ denotes observed efforts. Table 9 reports the average SSD in each treatment. Column 2 reports the average SSD_i^{BNE} and Column 3 reports the average SSD_i^{BLF}. We find that, in the Sim-G3 treatment, the average SSD_i^{BNE} is statistically significantly higher than SSD_i^{BLF}, indicating that our proposed overconfidence model fits the data significantly better than BNE predictions.

Table 9: Overview of the Sum of Squared Deviations (SSD)

	Average SSD based on		Wilcoxon Signed rank test
Treatment	BNE	BLF	One-sided p values
Sim-G2	3912.5	3912.5	0.36
Sim-G3	814.8	678.9	0.035
Seq-G2	806.0	815.2	0.135
Seq-G3	340.4	337.8	0.36

Moving on to sequential contests, the individual behaviors there show a different pattern. Although the first players in both Seq-G2 and Seq-G3 also overestimate their winning probabilities (Seq-G2: 4.74% higher than BNE, $p = 0.034$; Seq-G3: 12.65% higher than BNE, $p = 0.034$, one-sided signed rank tests) and the estimated $\widetilde{c}$ in the misperceived winning probability is significantly lower than $c = 0.25$ (Column 3 and 4 in Table 8), they do not significantly over- or under-invest their effort, as shown in Table 7 (Column 3 and 4) earlier.[25] This suggests that overconfidence does not lead to high ability contestants' under-investment in sequential treatments. The reason, we speculate, is due to high ability contestants' raising their effort to compensate for their positional disadvantages (recall that here we only examine the first players in sequential contests and according to Proposition 1, they face substantial positional disadvantages).

Last, we checked the middle players' behaviors in the Seq-G3 treatment. We find that these players exert higher effort than predicted by their Best Response (BR) functions, and this is particularly true for high ability players. Recall that depending on the first player's actual effort, a middle player can choose one of the three strategies: fold (make zero effort), call (match the first player's effort), or raise (make an effort higher than the first player's).[26] We compute δ, the difference between their actual effort and the BR predictions. For players who should fold according to their BR predictions, their mean deviation, $\bar{\delta}$, is 1.36 and is statistically significantly higher than zero ($p = 0.034$, one-sided one-sample signed rank tests). Of those who should call, many (33%) fold, resulting in an under-investment ($\bar{\delta} = -2.12$, $p = 0.072$, one-sided one-sample signed rank

[25]Consistently, results from Table 9 show that the BLF model does not fit the data significantly better than the BNE model.

[26]The observed first players' effort indicates that 25% of the middle players should fold, 9% should call, and 66% should raise.

tests).[27] When they should raise, however, they over-invest significantly ($\bar{\delta} = 3.27$, $p = 0.034$, one-sided one-sample signed rank tests). These behaviors suggest that sequential contests are fairly complex games, even with a small number of players.

Summarizing our entire individual level analysis, we find some over-investment in effort in sequential contests, although most participants do not deviate far from their BNE predictions, except for those high ability contestants in the Sim-G3 treatment. We construct a simple behavioral model of overconfidence to explain why these Sim-G3 players under-invest their effort as opposed to over-investing, as predicted by models of risk-aversion. The overconfidence model fits our data and rationalizes the observed behavior.

6 Conclusion

In this study, by modeling crowdsourcing contests as n-player all-pay auctions with incomplete information, we analytically show that simultaneous (blind) contests are superior to sequential (open) contests in generating better highest quality solutions and in producing more efficient outcomes. We also show, somewhat counterintuitively, that simultaneous contests are not necessarily always more wasteful than sequential contests. We also conduct a laboratory experiment which provides supporting evidence for these theoretical predictions. Additionally, our experimental data show that the efficiency of sequential contests decreases in the number of contestants, which further reduces their performance relative to simultaneous contests.

Our contributions are two-fold. First, compared to the existing all-pay auction and contest literature which mainly focuses on simultaneous games, this study investigates the impact of players' bidding order on the performance of individual players in sequential contests, and more importantly, it analytically compares simultaneous and sequential contests under incomplete information. While prior research has discovered cases in which favoritism (i.e., favoring some individual participants) can improve the quality of the best solutions (Chawla et al., 2012), our study shows that, as a specific case of favoritism (i.e., favoring late players), sequential contests actually produce lower expected highest quality than contests without favoritism (i.e., simultaneous contests).

Second, this is the first study to experimentally compare simultaneous and sequential contests in a setting with incomplete information. Although prior studies using field experiments have compared mechanisms resembling these two contests, our laboratory experiment is the first to empirically test theoretical predictions derived from game theoretical analysis with incomplete information. Our study complements these prior empirical studies by analyzing the same problem in a highly abstracted environment that strips away idiosyncratic factors of actual crowdsourcing platforms.

These findings have straightforward implications for practitioners of crowdsourcing. If the solution quality for a task depends primarily on the solver's ability and effort, the contest holder

[27] The difference becomes statistically nonsignificant in the second half of the experiment (mean difference $= -1.75$, $p = 0.233$, one-sided one-sample signed rank tests).

is better off choosing a simultaneous over a sequential contest, in order to profit from the best solution. A social planner (e.g., a crowdsourcing contest platform) may also prefer simultaneous contests since they produce more efficient outcomes, and do not necessarily waste more effort among all participants. For problems that involve high innovation, sequential contests may be considered together with their negative impacts on participants' incentives for putting forth effort.

Certainly, our simple model cannot capture all the important dynamics in crowdsourcing, e.g., in creative or innovative tasks, a solver's output could be influenced by factors beyond her effort or ability, e.g., inspiration (Wooten and Ulrich, 2013) or collaboration (Boudreau and Lakhani, 2014). In future work, for realism, a number of modeling assumptions also need to be relaxed. These assumptions include our specific distributional assumption for contestants' ability factors (as highlighted by Yang et al., 2011), the implicit assumption of the one-entry-per-participant rule and exogenous entry timing (as highlighted by Bockstedt et al., 2014). Task uncertainty could also be an influencer of crowdsourcing participants' strategic decisions (Boudreau et al., 2011), and explicitly modeling such uncertainties could provide guidance to choosing between sequential and simultaneous contests based on problem characteristics. Ultimately, theoretical predictions about the comparative performances of these two mechanisms can be tested using field experiments (Harrison and List, 2004; Benz and Meier, 2008), while systematically varying factors such as task uncertainty and participant ability distributions.

7 Acknowledgements

We thank Juan Carrillo and Isabelle Brocas for providing us with access to the Los Angeles Behavioral Economics Laboratory (LABEL). We would also like to thank Yan Chen, Jeffrey Mackie-Mason, Lijia Wei and seminar participants at the 2013 North American ESA meetings, the 2013 Annual Xiamen University International Workshop on Experimental Economics, and the 2014 Asian-Pacific ESA meetings for helpful discussions and comments, and Chao Tang for excellent research assistance. The financial support from Tsinghua University is gratefully acknowledged. Lian Jian also thanks the financial support from the APOC program at the Annenberg School of Communication, University of Southern California.

References

Alicke, M. D., M. L. Klotz, D. L. Breitenbecher, T. J. Yurak, and D. S. Vredenburg, "Personal Contact, Individuation, and the Better-than-Average Effect," *Journal of Personality and Social Psychology*, 2004, *68* (5), 804–825.

Amann, E. and W. Leininger, "Asymmetric All-Pay Auctions with Incomplete Information: The Two-Player Case," *Games and Economic Behavior*, 1996, *14*, 1–18.

Amegashie, J. A., C. B. Cadsby, and Y. Song, "Competitive Burnout: Theory and Experimental Evidence," *Games and Economic Behavior*, 2007, *59*, 213–239.

Araujo, R. M., "99designs: An Analysis of Creative Competition in Crowdsourced Design," in "Proceedings of the First AAAI Conference on Human Computation and Crowdsourcing" Palm Springs, CA 2013, pp. 17–24.

Archak, N. and A. Ghose, "Learning-by-doing and project choice: a dynamic structural model of crowdsourcing," *In International Conference on Information Systems (ICIS) 2010 Proceedings*, 2010.

_ **and A. Sundararajan**, "Attracting the best and the brightest: asymptotics and optimal prize structure of all-pay contests with heterogeneous participants," *Working paper*, 2009.

Augenblick, N., "Consumer and Producer Behavior in the Market for Penny Auctions: A Theoretical and Empirical Analysis," *Working paper*, 2014.

Barut, Y., D. Kovenock, and C. Noussair, "A Comparison of Multiple-Unit All-Pay and Winner-Pay Auctions Under Incomplete Information," *International Economic Review*, 2002, *43* (3), 675–708.

Benz, M. and S. Meier, "Do People Behave in Experiments as in the Field?-Evidence from Donations," *Experimental Economics*, 2008, *11* (3), 268–281.

Bockstedt, J., C. Druehl, and A. Mishra, "Heterogeneous Problem-Solving Behavior and its Implications for Success in Unblind Innovation Contests," *Working paper*, 2014.

Boudreau, K. J. and K. R. Lakhani, "Cumulative Innovation & Open Disclosure of Intermediate Results: Evidence from a Policy Experiment in Bioinformatics," *Working paper*, 2014.

_ **, C. E. Helfat, K. R. Lakhani, and M. Menietti**, "Performance Responses to Competition Across Skill-Levels in Rank Order Tournaments: Field Evidence and Implications for Tournament Design," *Working paper*, 2013, pp. 1–41.

_ **, N. Lacetera, and K. R. Lakhani**, "Incentives and Problem Uncertainty in Innovation Contests: An Empirical Analysis," *Management Science*, 2011, *57* (5), 843–863.

Brabham, D. C., "Moving the crowd at Threadless," *Information, Communication & Society*, 2010, *13* (8), 1122–1145.

Carpenter, J., J. Holmes, and P.H. Matthews, "An Introduction to "Bucket Auctions" for Charity," *Working paper*, 2011.

Chawla, S. and J. D. Hartline, "Auctions with unique equilibria," *Proceedings of the fourteenth ACM conference on electronic commerce*, 2013, pp. 181–196.

— , — , **and B. Sivan**, "Optimal crowdsourcing contests," *Proceedings of the Twenty-Third Annual ACM-SIAM Symposium on Discrete Algorithms*, 2012, pp. 856–868.

Che, Y.-K. and I. Gale, "Optimal Design of Research Contests," *American Economic Review*, 2003, *93* (3), 646–671.

Dahan, E. and H. Mendelson, "An extreme-value model of concept testing," *Management Science*, 2001, *47* (1), 102–116.

Davis, D. D. and R. J. Reilly, "Do Too Many Cooks Spoil the Stew? An Experimental Analysis of Rent-seeking and the Role of a Strategic Buyer," *Public Choice*, 1998, *95*, 89–115.

Dechenaux, E., D. Kovenock, and R. M. Sheremeta, "A Survey of Experimental Research on Contests, All-Pay Auctions and Tournaments," 2014. Forthcoming, Experimental Economics.

DiPalantino, D. and M. Vojnovic, "Crowdsourcing and All-Pay Auctions," in "Proceedings of the tenth ACM conference on Electronic commerce (EC)" ACM New York, NY 2009, pp. 119–128.

Duffy, J. and A. Matros, "All-Pay Auctions vs. Lotteries as Provisional Fixed-Prize Fundraising Mechanisms: Theory and Evidence," 2013. Working paper.

Fibich, G., A. Gavious, and A. Sela, "All-pay Auctions with Risk-averse Players," *International Journal of Game Theory*, 2006, *34* (4), 583–599.

Fischbacher, U., "z-Tree: Zurich Toolbox for Ready-made Economic Experiments," *Experimental Economics*, 2007, *10* (2), 171–178.

Fullerton, R. L. and R. P. McAfee, "Auctioning Entry into Tournaments," *The Journal of Political Economy*, 1999, *107* (3), 573–605.

Glazer, A. and R. Hussin, "Optimal contests," *Economic Inquiry*, 1988, *26* (1), 133–143.

Gneezy, U. and R. Smorodinsky, "All-pay Auction: An Experimental Study," *Journal of Economic Behavior and Organization*, 2006, *61*, 255–275.

Grosskopf, B., L. Rentschler, and R. Sarin, "Asymmetric Information in Contests: Theory and Experiments," *Working paper*, 2010.

Harhoff, D., J. Henkel, and E. von Hippel, "Profiting from voluntary information spillovers: how users benefit by freely revealing their innovations," *Research Policy*, 2003, *32* (10), 1753–1769.

Harrison, G. W. and J. A. List, "Field Experiments," *Journal of Economic Literature*, December 2004, *42* (4), 1009–1055.

Hillman, A. L. and J. G. Riley, "Politically contestable rents and transfers," *Economics and Politics*, 1989, *1* (1), 17–40.

Huang, Y., P. V. Singh, and T. Mukhopadhyay, "Crowdsourcing Contests: A Dynamic Structural Model of the Impact of Incentive Structure on Solution Quality," *Working paper*, 2014.

Jeppesen, L. B. and K. R. Lakhani, "Marginality and Problem-Solving Effectiveness in Broadcast Search," *Organization Science*, 2010, *21* (5), 1016–1033.

Kirkegaard, R., "Favoritism in asymmetric contests: Head starts and handicaps," *Games and Economic Behavior*, 2012, *76* (1), 226–248.

Konrad, K. A., *Strategy and Dynamics in Contests*, New York, NY: Oxford University Press, 2009.

Krishna, V. and J. Morgan, "An Analysis of the War of Attrition and the All-Pay Auction," *Journal of Economic Theory*, 1997, *72* (2), 343–362.

Liu, T. X., "Endogenous Sequential All-Pay Auctions: An Experimental Study," 2013. Working paper.

_ , J. Yang, L. A. Adamic, and Y. Chen, "Crowdsourcing with All-pay Auctions: a Field Experiment on Taskcn," 2014. Forthcoming, Management Science.

Mermer, A. G., "Contests with expectation-based loss-averse players," 2013. Working paper.

Moldovanu, B. and A. Sela, "The optimal allocation of prizes in contests," *American Economic Review*, 2001, *91* (3), 542–558.

_ and _ , "Contest Archiecture," *Journal of Economic Theory*, 2006, *126*, 70–96.

Moore, D. A. and P. J. Healy, "The trouble with overconfidence," *Psychological Review*, 2008, *115* (2), 502–517.

Morgan, J., "Sequential Contests," *Public Choice*, 2003, *116* (1), 1–18.

Muller, W. and A. Schotter, "Workaholics and dropouts in organizations," *Journal of the European Economics Association*, 2010, *8* (4), 717–743.

Myerson, R. B., "Optimal Auction Design," *Mathematics of operations research*, 1981, *6* (1).

Noussair, C. and J. Silver, "Behavior in all-pay auctions with incomplete information," *Games and Economic Behavior*, 2006, *55* (1), 189–206.

Nyarko, Y. and A. Schotter, "An Experimental Study of Belief Learning Using Elicited Beliefs," *Econometrica*, 2002, *70* (3), 971–1005.

Parreiras, S. O. and A. Rubinchik, "Contests with three or more heterogeneous agents," *Games and Economic Behavior*, 2010, *68*, 703–715.

Plott, C. and V. Smith, "An Experimental Examination of Two Exchange Institutions," *Review of Economic Studies*, 1978, *45*, 133–153.

Potters, J., C. G. de Vries, and F. van Winden, "An Experimental Examination of Rational Rent-Seeking," *European Journal of Political Economy*, 1998, *14*, 783–800.

Segev, E. and A. Sela, "Multi-Stage Sequential All-Pay Auctions," *Working paper*, 2014.

_ **and** _ , "Sequential all-pay auctions with head starts," *Social Choice and Welfare*, 2014.

_ **and** _ , "Sequential all-pay auctions with noisy outputs," *Journal of Mathematical Economics*, 2014, *50*, 251–261.

Sheremeta, R., Masters W., and T. Cason, "Entry into Winner-Take-All and Proportional-Prize Contests: An Experimental Study," *Journal of Public Economics*, 2010, *94*, 604–611.

Silverman, R. E., "Big firms try crowdsourcing," *The Wall Street Journal*, Jan 17, 2012. Available at: http://online.wsj.com/article/SB10001424052970204409004577157493201863200.html 2012.

Taylor, C. R, "Digging for golden carrots: an analysis of research tournaments," *American Economic Review*, 1995, *85* (4), 872–890.

Terwiesch, C. and Y. Xu, "Innovation Contests, Open Innovation, and Multiagent Problem Solving," *Management Science*, 2008, *54* (9), 1529–1543.

Tullock, G., *Efficient Rent Seeking*, College Station, Tx: Texas A&M University Press,

von Hippel, E., *Democratizing Innovation*, Cambridge, MA: MIT Press, 2005.

Walter, T. and A. Back, "Towards Measuring Crowdsourcing Success: An Empirical Study on Effects of External Factors in Online Idea Contest," in "Proceedings of the 6th Mediterranean Conference on Information Systems (MCIS)" Limassol, Cyprus 2011.

Weber, R. J., "Auctions and competitive bidding," in H. Peyton Young, ed., *Fair Allocation, American Mathematical Society Proceedings of Symposia in Applied Mathematics*, Vol. 33, Providence, Rhode Island: American Mathematical Society, 1985, pp. 143–170.

Wooten, J. O. and K. T. Ulrich, "Idea Generation and the Role of Feedback: Evidence from Field Experiments with Innovation Tournaments," *SSRN Electronic Journal*, 2011.

_ **and** _ , "The Impact of Visibility in Innovation Tournaments: Evidence from Field Experiments," *Working paper*, 2013.

Yang, J., L. A. Adamic, and M. S. Ackerman, "Crowdsourcing and Knowledge Sharing: Strategic User Behavior on Taskcn," in "Proceedings of the 9th ACM Conference on Electronic Commerce (EC'08)" 2008.

Yang, Y., P.-Y. Chen, and P. Pavlou, "Open innovation: an empirical study of online contests," in "International Conference on Information Systems (ICIS) 2009 Proceedings" 2009.

_ , _ , and R. Banker, “Winner Determination of Open Innovation Contests in Online Markets,” in “International Conference on Information Systems (ICIS) 2011 Proceedings” 2011.

Online Appendices

A Probability of Winning

We first prove Proposition 1.

Proof. Consider two contestants i and j ($j > i$) with ability factors a_i and a_j respectively. We only consider the cases in which either i or j wins the contest, and see if j has a higher chance of winning than i. As the distribution of a_i and a_j are *i.i.d.*, both have an equal chance to be larger.

First consider the case when $a_j > a_i$. If the winner is between i and j, it must be that $q_j \geq q_i$. Note that here $q_j = q_i$ is a shorthand for $q_j = q_i + \epsilon$, which would imply that the later player outperforms the earlier one. If player i folds, i.e., $q_i = 0$, then by definition $q_j \geq 0$, so $q_j \geq q_i$ holds. If player i raises, i.e., $q_i = q_i^*$, player j will either call or raise, since $a_j > (q_i^*)^{d_j}$ (due to $a_j > a_i$). Last, if player i calls, i.e., $q_i = \max\{q_k(a_k)\}_{k<i}$, then j will also either call or raise, again due to $a_j > a_i$.

Next, we consider the opposite case in which $a_i > a_j$. If $q_i = q_i^*$, then as long as $a_j > (q_i^*)^{d_j}$, player j will either call or raise. This is possible as we can easily verify that $(q_i^*)^{d_j} < a_i$. If $q_i = \max\{q_k(a_k)\}_{k<i}$, then $[\max\{q_k(a_k)\}_{k<i}]^{d_i} < a_i$. Since $d_j > d_i$, we arrive at $[\max\{q_k(a_k)\}_{k<i}]^{d_j} < a_i$, which shows that $a_j > [\max\{q_k(a_k)\}_{k<i}]^{d_j}$ is possible, in which case j will either call or raise. □

Lemma 1 provides the closed-form solution for the winning probability of player i.

Lemma 1. *The expected probability of winning of player i in an n-player sequential contest is the sum of the following three items, A, B, and C:*

$$\begin{aligned} A &= \sum_{m=1}^{i-1} \sum_{l=m+1}^{i} \frac{d_m}{\prod_{k=1}^{l-1}(1-d_k)^c} \Big[\frac{(1-d_{l-1})^{\frac{c}{d_{l-1}} \sum_{k=1,k\neq l\sim i}^{n} d_k} - (1-d_l)^{\frac{c}{d_l} \sum_{k=1,k\neq l\sim i}^{n} d_k}}{\sum_{k=1,k\neq l\sim i}^{n} d_k} \\ &\quad - \frac{(1-d_{l-1})^{\frac{c}{d_{l-1}} \sum_{k=1,k\neq l\sim i-1}^{n} d_k} - (1-d_l)^{\frac{c}{d_l} \sum_{k=1,k\neq l\sim i-1}^{n} d_k}}{\sum_{k=1,k\neq l\sim i-1}^{n} d_k} \Big] \end{aligned}$$

$$B = \sum_{m=1}^{i-1} \frac{d_m (1-d_i)^{\frac{c}{d_i} \sum_{k=1}^{n} d_k} (1 - (1-d_i)^c)}{(\prod_{k=1}^{i}(1-d_k)^c) \sum_{k=1}^{n} d_k}$$

$$C = \frac{d_i (1-d_i)^{\frac{c}{d_i} \sum_{k=1}^{n} d_k}}{(\prod_{k=1}^{i}(1-d_k)^c) \sum_{k=1}^{n} d_k}$$

Note, the notation $\sum_{k=1,k\neq l\sim i}^{n} d_k$ means $\sum_{k=1}^{n} d_k - \sum_{k=l}^{i} d_k$.

Proof. For a given player i to win, there are only three scenarios:

A. There is an existing highest effort initially made by player m ($m < i$). $q_m = q_m^*$ and $\frac{(q_m^*)^{d_i}}{1-d_i} > 1$. Player i wins by calling, thus $q_i = q_m^* + \epsilon$. For any player j ($j > i$), $a_j < (q_m^*)^{d_j}$.

B. There is an existing highest effort initially made by player m ($m < i$). $q_m = q_m^*$, $\frac{(q_m^*)^{d_i}}{1-d_i} < 1$, and $(q_m^*)^{d_i} < a_i < \frac{(q_m^*)^{d_i}}{1-d_i}$. In this case, player i wins by calling, thus $q_i = q_m^* + \epsilon$. For any player j ($j > i$), $a_j < (q_m^*)^{d_j}$.

C. Player i's q_i^* is higher than any previous player's effort, such that player i wins by raising, thus $q_i = q_i^*$. For any player j ($j > i$), $a_j < (q_i^*)^{d_j}$.

We use A (B, C) to denote the probability of player i winning under the scenario A (B, C). For ease of exposition, let us define $\widehat{d_a d_b} = \frac{(1-d_a)^{\frac{d_b}{d_a}}}{1-d_b}$, which implies $\widehat{d_n d_b} = 0$. So $A = \sum_{m=1}^{i-1} \sum_{l=m+1}^{i} \int_{\widehat{d_l d_m}}^{\widehat{d_{l-1} d_m}} (\prod_{k=1,k\neq m}^{l-1} P(q_k^* < q_m^*)) P(a_i > (q_m^*)^{d_i}) \prod_{j=i+1}^{n} P(a_j < (q_m^*)^{d_j}) dF(a_m)$. $B = \int_0^{d_i d_m} \prod_{k=1,k\neq m}^{i-1} P(q_k^* < q_m^*) P((q_m^*)^{d_i} < a_i < \frac{(q_m^*)^{d_i}}{1-d_i}) \prod_{j=i+1}^{n} P(a_j < (q_m^*)^{d_j}) dF(a_m)$. $C = \int_0^1 \prod_{k=1}^{i-1} P(q_k^* < q_i^*) \prod_{j=i+1}^{n} P(a_j < (q_i^*)^{d_j}) dF(a_i)$.

□

B Proof for Proposition 2

Proof. We define a function $q(x; a) = (a(1-x))^{\frac{1}{x}}$, by letting $x = (1-c)^{n-i}$. Note that x is a monotonically increasing function of i ($1 \leq i \leq n$).

$$q'(x; a) = -\frac{1}{x^2}(1-x)^{\frac{1}{x}} a^{\frac{1}{x}} \ln a + a^{\frac{1}{x}}(1-x)^{\frac{1}{x}}(-\frac{\ln(1-x)}{x^2} - \frac{1}{x(1-x)})$$

The sign of $q'(x; a)$ is the same as the sign of $-\ln a - \ln(1-x) - \frac{x}{1-x}$. Let $f(x) = \ln(1-x) + \frac{x}{1-x}$, then $f'(x) = \frac{x}{(1-x)^2} > 0$. We also have $-\ln a \in (0, \infty)$, $f(0) = 0$, and $f(1) = \infty$. So for any given $a \in (0, 1)$, there exists a unique x^*, such that $q'(x^*; a) = 0$; $q'(x; a) > 0$ if $x < x^*$; $q'(x; a) < 0$ if $x > x^*$. So $q(x; a)$ will increase until $x = x^*$, then decrease; in other words, $q(x; a)$ attains its global maximum at x^*. Setting $q'(x; a) = 0$ and the FOC is equivalent to

$$-\ln a - \frac{x^*}{(1-x^*)} - \ln(1-x^*) = 0 \tag{4}$$

Given that the solution to the FOC is unique, every a uniquely determines an x^*. An explicit functional form for $x^*(a)$, however, cannot be obtained, so we apply the implicit function theorem to Equation (4), and obtain $\frac{\mathrm{d}x^*}{\mathrm{d}a} = -\frac{(1-x^*)^2}{ax^*} < 0$.

Since we treat i as a continuous variable, i.e. $i \in [1, n-1]$ (Note that $q(1; a) = 0 < q(1-\epsilon; a)$ implies that $\hat{i}$ must be less than n), then $x \in [(1-c)^{n-1}, 1-c]$. We need to consider three scenarios, $x^* < (1-c)^{n-1}$, $(1-c)^{n-1} \leq x^* \leq 1-c$ and $x^* > 1-c$.

If $x^* < (1-c)^{n-1}$, we have $\hat{i} = 1$, then $\frac{\partial \hat{i}}{\partial n} = 0$, $\frac{\partial \hat{i}}{\partial c} = 0$ and $\frac{\partial \hat{i}}{\partial a} = \frac{\partial \hat{i}}{\partial x^*}\frac{\mathrm{d}x^*}{\mathrm{d}a} = 0$.

If $(1-c)^{n-1} < x^* < 1-c$, we have $x^* = (1-c)^{n-\hat{i}}$, which means that $\hat{i}$ is a function of x^*, c and n. By implicit function theorem we know $\frac{\partial \hat{i}}{\partial c} > 0$, $\frac{\partial \hat{i}}{\partial n} > 0$ and $\frac{\partial \hat{i}}{\partial a} = \frac{\partial \hat{i}}{\partial x^*}\frac{\mathrm{d}x^*}{\mathrm{d}a} < 0$. .

If $x^* > 1-c$, we have $\hat{i} = n-1$, then $\frac{\partial \hat{i}}{\partial n} = 1 > 0$, $\frac{\partial \hat{i}}{\partial c} = 0$ and $\frac{\partial \hat{i}}{\partial a} = \frac{\partial \hat{i}}{\partial x^*}\frac{\mathrm{d}x^*}{\mathrm{d}a} = 0$.

Given these three scenarios, it is straightforward to check when $x^* = (1-c)^{n-1}$ or $x^* = 1-c$ we have $\frac{\partial \hat{i}}{\partial c} \geq 0$, $\frac{\partial \hat{i}}{\partial n} \geq 0$ and $\frac{\partial \hat{i}}{\partial a} \leq 0$. □

C Proof for Proposition 3

Proof.

$$\frac{\partial EHQ^{Sim}}{\partial n} = \frac{(2n-1)c^2(1+(n-1)c)(2nc-c+1)-n(n-1)c^2(c(2nc-c+1)+2c(1+(n-1)c))}{(1+(n-1)c)^2(2nc-c+1)^2} \quad (5)$$

Therefore, it is equivalent to showing the following:

$$(2n-1)c^2(1+(n-1)c)(2nc-c+1) - n(n-1)c^2(c(2nc-c+1)+2c(1+(n-1)c)) > 0$$

This can be simplified to:

$$F(n) \equiv (3n^2-4n+2)c - (n-1)^2c^2 + 2n - 1 > 0$$

As $F(2) > 0$, and $F'(n) = (6n-4)c - 2(n-1)c^2 + 2 > 0$, therefore, $F(n) \equiv (3n^2-4n+2)c - (n-1)^2c^2 + 2n - 1 > 0$ always hold.

To show EHQ^{Sim} increases in c, we need to show

$$\frac{\partial EHQ^{Sim}}{\partial c} = v\frac{2cn(n-1)(1+(n-1)c)((2n-1)c+1)-(n^2-n)c^2(2(2n^2-3n+1)c+3n-2)}{(1+(n-1)c)^2(2nc-c+1)^2}. \quad (6)$$

It is equivalent to show that $2cn(n-1) + c^2n(n-1)(3n-2) > 0$, which is straightforward. □

D Proof for Proposition 4

Proof. We start by deriving the expected highest bid if bidder i's bid is the highest, denoted by EHQ_i^{Seq}. For bidder i's bid to be the highest, first, it has to be that any preceding bidder k's ($k \in \{1, \ldots, i-1\}$) bid, q_k, satisfies $\frac{q_k^{d_i}}{1-d_i} \leq a_i$.

Second, for q_i to be the highest, all bidders after bidder i, thus bidder j ($j \in \{i+1, \ldots, n\}$), have to bid zero or q_i ($q_i + \epsilon$, to be exact). Recall that bidder j's equilibrium bidding function has three parts. The third part does not exist if $\frac{q_i^{d_j}}{1-d_j} \geq 1$, in which case, bidder j only has the choices of bidding zero or q_i, regardless of the value of a_j. For $\frac{q_i^{d_j}}{1-d_j} \geq 1$ to hold for bidder j, a_i has to be in the range of $[\frac{(1-d_j)^{\frac{d_i}{d_j}}}{1-d_i}, 1]$.

Further, since $(1-d_j)^{\frac{1}{d_j}}$ is a decreasing function of j, if $\frac{q_i^{d_j}}{1-d_j} \geq 1$ is satisfied for bidder j, it is satisfied for all bidders after j. So to account for all possibilities, we start by integrating a_i over $[\frac{(1-d_{i+1})^{\frac{d_i}{d_{i+1}}}}{1-d_i}, 1]$, the range in which the value of q_i will make all subsequent bidders choose either zero or q_i, followed by integrating a_i over the next lower range, $[\frac{(1-d_{i+2})^{\frac{d_i}{d_{i+2}}}}{1-d_i}, \frac{(1-d_{i+1})^{\frac{d_i}{d_{i+1}}}}{1-d_i}]$, and so on. Following the notations used in Lemma 1, we again define $\widehat{d_a d_b} = \frac{(1-d_a)^{\frac{d_b}{d_a}}}{1-d_b}$, which implies $\widehat{d_n d_b} = 0$.

Taken together,

$$\begin{aligned}
\text{EHQ}_i^{Seq} &= \sum_{j=i+1}^{n} \int_{\widehat{d_j d_i}}^{\widehat{d_{j-1} d_i}} q_i \prod_{k=1}^{i-1} P\left(\frac{q_k^{d_i}}{1-d_i} < a_i\right) \prod_{m=i+1}^{j-1} P\left(a_m < \frac{q_i^{d_m}}{1-d_m}\right) f(a_i) da_i \\
&= \sum_{j=i+1}^{n} \int_{\widehat{d_j d_i}}^{\widehat{d_{j-1} d_i}} q_i \prod_{k=1, k\neq i}^{j-1} \left(\frac{(a_i(1-d_i))^{\frac{d_k}{d_i}}}{1-d_k}\right)^c f(a_i) da_i \\
&= \sum_{j=i+1}^{n} \int_{\widehat{d_j d_i}}^{\widehat{d_{j-1} d_i}} q_i a_i^{-c} \prod_{k=1}^{j-1} \left(\frac{(a_i(1-d_i))^{\frac{d_k}{d_i}}}{1-d_k}\right)^c c a_i^{c-1} da_i \\
&= \sum_{j=i+1}^{n} \frac{c(1-d_i)^{\frac{1}{d_i}}}{\prod_{m=1}^{j-1}(1-d_m)^c} \int_{\widehat{d_j d_i}}^{\widehat{d_{j-1} d_i}} a_i^{\frac{1}{d_i}-1} (a_i(1-d_i))^{\frac{c}{d_i}\sum_{k=1}^{j-1} d_k} da_i \\
&= \sum_{j=i+1}^{n} \frac{c(1-d_i)^{\frac{1}{d_i}}}{\prod_{m=1}^{j-1}(1-d_m)^c} \int_{\widehat{d_j d_i}}^{\widehat{d_{j-1} d_i}} a_i^{\frac{1}{d_i}-1} (a_i(1-d_i))^{\frac{c}{d_i}\frac{(1-c)^n}{c}\left(\frac{1}{(1-c)^{j-1}}-1\right)} da_i \\
&= \sum_{j=i+1}^{n} \frac{c\left((1-d_{j-1})^{\frac{1+(1-c)^{n-j+1}-(1-c)^n}{d_{j-1}}} - (1-d_j)^{\frac{1+(1-c)^{n-j+1}-(1-c)^n}{d_j}}\right)}{\left(\prod_{m=1}^{j-1}(1-d_m)^c\right)\left(\frac{1+c\sum_{m=1}^{j-1} d_m}{d_i}\right)}
\end{aligned} \tag{7}$$

Note that the above derivation uses the formula for the sum of a geometric series, i.e., $\sum_{k=1}^{j-1} d_k = (1-c)^n \sum_{k=1}^{j-1} \frac{1}{(1-c)^k} = \frac{(1-c)^n}{c}\left(\frac{1}{(1-c)^{j-1}} - 1\right)$.

Summing up EHQ_i^{Seq} over all $i \in \{1, \cdots, n\}$, we obtain

$$\begin{aligned}
\text{EHQ}^{Seq} &= \sum_{i=1}^{n}\sum_{j=i+1}^{n} \frac{c\left((1-d_{j-1})^{\frac{1+(1-c)^{n-j+1}-(1-c)^{n}}{d_{j-1}}} - (1-d_j)^{\frac{1+(1-c)^{n-j+1}-(1-c)^{n}}{d_j}}\right)}{(\prod_{m=1}^{j-1}(1-d_m)^c)(\frac{1+c\sum_{m=1}^{j-1}d_m}{d_i})} \\
&= \sum_{i=1}^{n}\sum_{j=i+1}^{n} \frac{c\left((1-d_{j-1})^{\frac{1+(1-c)^{n-j+1}-(1-c)^{n}}{d_{j-1}}} - (1-d_j)^{\frac{1+(1-c)^{n-j+1}-(1-c)^{n}}{d_j}}\right)}{(\prod_{m=1}^{j-1}(1-d_m)^c)(\frac{1+(1-c)^{n-j+1}-(1-c)^{n}}{d_i})} \quad (8)
\end{aligned}$$

As n increases to infinity, EHQ^{Seq} has an upper bound of $\frac{1}{e}$. This can be easily seen since $EHQ^{Seq} < (1-(1-c)^{n-1})^{\frac{1}{(1-c)^{n-1}}}$ and $\lim_{n\to\infty}(1-(1-c)^{n-1})^{\frac{1}{(1-c)^{n-1}}} = \frac{1}{e}$. This upper bound is not tight. □

E Proof for Theorem 1

Proof. Due to the complexity of the expression for EHQ^{seq}, standard methods involving directly taking derivatives do not work. We start by simplifying the inequality we aim to prove. First, since the last bidder never raises, we have

$$HQ^{Seq}(n,c) = max\{(a_i(1-d_i))^{\frac{1}{d_i}} | i = 1, 2, \cdots, n-1\}$$

which implies,

$$HQ^{Seq}(n,c) \le (1-d_1)^{\frac{1}{d_1}}(max\{a_i | i = 1, 2, \cdots, n-1\})^{\frac{1}{d_{n-1}}} \quad (9)$$

$$HQ^{Seq}(n,c) \le (1-d_1)^{\frac{1}{d_1}}(max\{a_i | i = 1, 2, \cdots, n-1, n\})^{\frac{1}{d_{n-1}}} \quad (10)$$

Given that $F(a) = a^c$, the distribution of the highest a_i out of $\{a_1, a_2, \cdots, a_n\}$ is $f_{a(n)}(a) = na^{nc-c}ca^{c-1} = nca^{nc-1}$, where $f_{a(n)}(a)$ denotes the n^{th} order statistic of a. Therefore, Equation (10) implies

$$\begin{aligned}
EHQ^{Seq}(n,c) &\le (1-(1-c)^{n-1})^{\frac{1}{(1-c)^{n-1}}} \int_0^1 a^{\frac{1}{1-c}} nca^{nc-1} da \\
&= (1-(1-c)^{n-1})^{\frac{1}{(1-c)^{n-1}}} \frac{nc}{nc + \frac{1}{1-c}} \quad (11)
\end{aligned}$$

Similarly, Equation (9) implies

$$EHQ^{Seq}(n,c) \leq (1-(1-c)^{n-1})^{\frac{1}{(1-c)^{n-1}}} \frac{(n-1)c}{(n-1)c+\frac{1}{1-c}} \tag{12}$$

The rest of the proof proceeds as follows. Starting with Equation (11), Steps $1 \sim 6$ prove that $EHQ^{Sim}(n,c) > EHQ^{Seq}(n,c)$ holds for $n \geq 5$. Starting with Equation (12), Step 7 proves that $EHQ^{Sim}(n,c) > EHQ^{Seq}(n,c)$ holds for $n < 5$. The reason why the proof has to be broken into two parts is because while Equation (11) is relatively easier to work with in proving the cases of $n \geq 5$, it cannot be used to prove the cases of $n < 5$ because it enlarges $EHQ^{Seq}(n,c)$ too much such that $(1-(1-c)^{n-1})^{\frac{1}{(1-c)^{n-1}}} \frac{nc}{nc+\frac{1}{1-c}} < EHQ^{Sim}(n,c)$ does not hold for $n < 5$.

Step 1. Show that inequality $\frac{Nc}{2Nc+c+1} > (1-(1-c)^N)^{\frac{1}{(1-c)^N}}$ ($N = n-1$) implies $EHQ^{Sim}(n,c) > EHQ^{Seq}(n,c)$.

Now, we re-write the expression for $EHQ^{Sim}(n,c)$ as $\frac{(n-1)c}{1+(n-1)c} \frac{nc+\frac{1}{1-c}}{2nc-c+1} \frac{nc}{nc+\frac{1}{1-c}}$. Comparing this expression and Equation (11), we know that if we could show $(1-(1-c)^{n-1})^{\frac{1}{(1-c)^{n-1}}} < \frac{(n-1)c}{1+(n-1)c} \frac{nc+\frac{1}{1-c}}{2nc-c+1}$, we would have shown $EHQ^{Sim}(n,c) > EHQ^{Seq}(n,c)$. Replacing $n-1$ with N, we obtain that if we could show that the following inequality holds for $N \geq 4$, we would have completed the proof (note that $\frac{nc+\frac{1}{1-c}}{1+(n-1)c} > 1$):

$$(1-(1-c)^N)^{\frac{1}{(1-c)^N}} < \frac{Nc}{2Nc+c+1} \tag{13}$$

Step 2. Show that if we could prove that Equation (14) has a unique solution of $c = 0$, we would have shown that inequality (13) holds.

$$(1-(1-c)^N)^{\frac{1}{(1-c)^N}} - \frac{Nc}{2Nc+c+1} = 0 \tag{14}$$

First, given that $N \geq 4$, when $c = 1$ we must have

$$\frac{Nc}{2Nc+c+1} - (1-(1-c)^N)^{\frac{1}{(1-c)^N}} = \frac{N}{2N+2} - \frac{1}{e} > 0.$$

If $\exists\, c \in (0,1)$, s.t.

$$\frac{Nc}{2Nc+c+1} - (1-(1-c)^N)^{\frac{1}{(1-c)^N}} < 0.$$

By continuity, $\exists\, \widetilde{c} \in (c,1)$ s.t.

$$\frac{N\widetilde{c}}{2N\widetilde{c}+\widetilde{c}+1} - (1-(1-\widetilde{c})^N)^{\frac{1}{(1-\widetilde{c})^N}} = 0,$$

which contradicts the assumption that Equation (14) has a unique solution.

Step 3. For any $N > 0$ and $t \in \{1, 2, \cdots\}$, we construct a sequence c_t^N that satisfies

the difference equation (Equation (15)), and show that, for any given N, proving that Equation (14) has a unique solution of $c = 0$ is equivalent to showing that the sequence c_t^N converges to zero, i.e., its infima is zero.

To construct the sequence c_t^N for any given N, we define a difference equation:

$$\frac{Nc_{t+1}}{2Nc_{t+1} + c_{t+1} + 1} - (1 - (1 - c_t)^N)^{\frac{1}{(1-c_t)^N}} = 0. \tag{15}$$

Solving c_{t+1} from Equation (15), we get

$$c_{t+1}(N, c_t) = \frac{(1 - (1 - c_t)^N)^{\frac{1}{(1-c_t)^N}}}{N - (2N + 1)(1 - (1 - c_t)^N)^{\frac{1}{(1-c_t)^N}}} \tag{16}$$

Define function $h(N, x) = \frac{x}{N-(2N+1)x}$, which is an increasing function of x. Define function $g(N, c) = (1 - (1 - c)^N)^{\frac{1}{(1-c)^N}}$, and we know $g(N, c)$ increases in c. Taken together, the properties of both function $h(N, x)$ and $g(N, c)$ imply that $\frac{\partial c_{t+1}(N,c_t)}{\partial c_t} > 0$.

Equation (15) induces a sequence $\{c_t^N\} =: \{c_t^N \text{ satisfy } (15) \mid c_0^N = 1\}$, which has two properties: 1) the sequence monotonically decreases; and 2) $c_t^N > 0$ for any $N > 0$ and any $t \geq 0$. We first establish property 1) by induction. Due to Equation (15) and $c_0^N = 1$, we know $c_1^N = \frac{1}{(e-2)N-1} < 1$, i.e. $c_1^N < c_0^N$. Given that $c_t^N < c_{t-1}^N$ and $\frac{\partial c_{t+1}(N,c_t)}{\partial c_t} > 0$, we know $c_{t+1}(N, c_t^N) < c_t(N, c_{t-1}^N)$, i.e. $c_{t+1}^N < c_t^N, \ \forall t \geq 0$. To establish property 2), since $\{c_t^N\}$ monotonically decreases, we know $c_t^N \leq 1$, which implies $0 \leq (1 - (1 - c_t^N)^N)^{\frac{1}{(1-c_t^N)^N}} \leq \frac{1}{e}, \forall t \geq 0$. It also should be noted that $h(N, x) \geq 0$, $\forall N \geq 4$, $\forall x \in (0, \frac{1}{e}]$. Therefore we have shown that $c_t^N > 0$ for any $N \geq 1$ and any $t \geq 0$.

Last, we show that the sequence $\{c_t^N\}$ converges to zero if and only if Equation (14) has a unique solution $c = 0$. To prove sufficiency, since $\{c_t^N\}$ is monotonic and bounded, it must converge. We denote its limit as $\widehat{c} = \lim_{t\to\infty} c_t^N$. Substituting $\{c_t^N\}$ into Equation (15) and taking limits on both sides, we arrive at

$$\frac{N\widehat{c}}{2N\widehat{c} + \widehat{c} + 1} - (1 - (1 - \widehat{c})^N)^{\frac{1}{(1-\widehat{c})^N}} = 0,$$

Given that this equation has a unique solution of zero, we know $\widehat{c} = 0$, i.e. the sequence $\{c_t^N\}$ converge to zero.

To prove necessity, suppose there exists another solution to Equation (14), i.e., a $\bar{c} \in (0, 1)$, s.t.

$$\frac{N\bar{c}}{2N\bar{c} + \bar{c} + 1} - (1 - (1 - \bar{c})^N)^{\frac{1}{(1-\bar{c})^N}} = 0,$$

it means $c_{t+1}(\bar{c}, N) = \bar{c}$ by (16). Because $\bar{c} \in (0, 1)$ and $\{c_t^N\}$ converges to zero, $\exists\, t,$ s.t. $c_t^N \geq \bar{c} > c_{t+1}^N$. But $c_t^N \geq \bar{c}$ and $\frac{\partial c_{t+1}(N,c_t)}{\partial c_t} > 0$ together imply $c_{t+1}(N, c_t^N) \geq c_{t+1}(N, \bar{c})$, i.e. $c_{t+1}^N \geq \bar{c}$, leading to a contradiction.

Taken together, given that $\{c_t^N\}$ monotonically decreases, if it converges to zero for any N, inequality (13) must hold. More precisely, if we can show that the infima of $\{c_t^N\}$ is zero for all $N \geq 4$, inequality (13) must hold. The next two steps (Step 4 and 5) establish this by induction.

Step 4. Show that for $N = 4, \forall c \in (0, 1)$, the infima of the sequence $\{c_t^N\}$ is zero.

In preparation for the proof, we first establish a few useful limits and an inequality:

$$\lim_{c \to 0} (1 - (1-c)^N) \ln (1 - (1-c)^N) = 0 \tag{17}$$

$$\lim_{c \to 0} (1 - (1-c)^N)^{\frac{1}{(1-c)^N} - 1} = 1 \tag{18}$$

$$x < -\ln (1 - x) < \frac{x}{1-x} \text{ ,} \forall x \in (0, 1) \tag{19}$$

To prove Equation (17),

$$\begin{aligned}
\lim_{c \to 0} (1 - (1-c)^N) \ln (1 - (1-c)^N) = & \quad \lim_{c \to 0} \frac{\ln (1 - (1-c)^N)}{(1 - (1-c)^N)^{-1}} \\
= & \quad \lim_{c \to 0} \frac{(1 - (1-c)^N)^{-1} N (1-c)^{N-1}}{-N(1-c)^{N-1}(1 - (1-c)^N)^{-2}} \\
= & \quad \lim_{c \to 0} -(1 - (1-c)^N) \\
= & \quad 0
\end{aligned}$$

To prove Equation (18),

$$\lim_{c \to 0} (1 - (1-c)^N)^{\frac{1}{(1-c)^N} - 1} = e^{\lim_{c \to 0} \frac{(1-(1-c)^N) \ln (1-(1-c)^N)}{(1-c)^N}} = 1 \text{ (by (17))}$$

To prove the inequalities in (19), we observe that when $x = 0$, $0 = -\ln (1 - 0) = \frac{0}{1-0}$. Taking derivatives with respect to x and note that $1 < \frac{1}{1-x} < \frac{1}{(1-x)^2}$, $\forall x \in (0, 1)$, it is straightforward to show that inequalities in (19) hold.

Next, we define function $f(N, c) = \frac{Nc}{2Nc+c+1}$, and recall that we have defined $g(N, c) = (1 - (1-c)^N)^{\frac{1}{(1-c)^N}}$. Then we derive their first, second, and third order partial derivatives with respect to c as follows,

$$
\begin{aligned}
\frac{\partial f(N,c)}{\partial c} &= \frac{N}{(2Nc+C+1)^2} \\
\frac{\partial^2 f(N,c)}{\partial c^2} &= \frac{-2N(2N+1)}{(2Nc+c+1)^3} \\
\frac{\partial^3 f(N,c)}{\partial c^3} &= \frac{6N(2N+1)^2}{(2Nc+c+1)^4} \\
\frac{\partial g(N,c)}{\partial c} &= (1-(1-c)^N)^{\frac{1}{(1-c)^N}}\left(\frac{N}{(1-c)(1-(1-c)^N)} + \frac{N\ln(1-(1-c)^N)}{(1-c)^{N+1}}\right) \\
\frac{\partial^2 g(N,c)}{\partial c^2} &= (1-(1-c)^N)^{\frac{1}{(1-c)^N}-1}\left(\frac{2N^2+N}{(1-c)^2} + \frac{N^2(1-(1-c)^N)(\ln(1-(1-c)^N))^2}{(1-c)^{2N+2}}\right. \\
&\quad \left. + \frac{2N^2\ln(1-(1-c)^N)}{(1-c)^{N+2}} + \frac{N(N+1)(1-(1-c)^N)\ln(1-(1-c)^N)}{(1-c)^{N+2}}\right).
\end{aligned}
$$

Observing that for any given N, we have $f(N,0) = g(N,0) = 0$, $\frac{\partial f(N,c)}{\partial c}|_{c=0} = \frac{\partial g(N,c)}{\partial c}|_{c=0} = N$ (by Equations (17), (18)), if we can find an open interval $(0, c(N))$, s.t. $\frac{\partial^2 f(N,c)}{\partial c^2} > \frac{\partial^2 g(N,c)}{\partial c^2}$, $\forall c \in (0, c(N))$, we must have $f(N,c) > g(N,c)$, $\forall c \in (0, c(N))$. In the following steps (a) $\sim$ (e), we will show that $c(N)$ exists for all N.

Step (a). By $\frac{\partial^3 f(N,c)}{\partial c^3} > 0$, we know $\frac{\partial^2 f(N,c)}{\partial c^2} > \frac{\partial^2 f(N,c)}{\partial c^2}|_{c=0} = -2N(2N+1)$, so $\frac{\partial^2 g(N,c)}{\partial c^2} < -2N(2N+1)$ would imply $\frac{\partial^2 f(N,c)}{\partial c^2} > \frac{\partial^2 g(N,c)}{\partial c^2}$.

Step (b). $\frac{\partial^2 g(N,c)}{\partial c^2} < -2N(2N+1) \Leftrightarrow (1-c)^2\frac{\partial^2 g(N,c)}{\partial c^2} < -(1-c)^2 2N(2N+1) \Leftarrow (1-c)^2\frac{\partial^2 g(N,c)}{\partial c^2} < -2N(2N+1)$. Thus, we just need to show $(1-c)^2\frac{\partial^2 g(N,c)}{\partial c^2} < -2N(2N+1)$.

Step (c). We observe, first, the last term of $\frac{\partial^2 g(N,c)}{\partial c^2}$ is less than zero. Second, by the inequalities in (19), we have $-1 < \frac{(1-(1-c)^N)\ln(1-(1-c)^N)}{(1-c)^N} < 0$, so $(1-(1-c)^N)^{\frac{1}{(1-c)^N}-1} = e^{\frac{(1-(1-c)^N)\ln(1-(1-c)^N)}{(1-c)^N}} \in (e^{-1}, 1)$, which implies $2N^2+N > (1-(1-c)^N)^{\frac{1}{(1-c)^N}-1}(2N^2+N)$. Utilizing these two observations, we have

$$
\begin{aligned}
&(1-c)^2\frac{\partial^2 g(N,c)}{\partial c^2} < 2N^2+N+(1-(1-c)^N)^{\frac{1}{(1-c)^N}-1} \\
&\left(\frac{N^2(1-(1-c)^N)(\ln(1-(1-c)^N))^2}{(1-c)^{2N}} + \frac{2N^2\ln(1-(1-c)^N)}{(1-c)^N}\right).
\end{aligned}
$$

If we can show $-2N(2N+1)$ is greater than the RHS of the above inequality, we would have shown $(1-c)^2\frac{\partial^2 g(N,c)}{\partial c^2} < -2N(2N+1)$. Collecting items and simplifying, we need to show the following,

$$-\frac{3(2N+1)}{N} > (1-(1-c)^N)^{\frac{1}{(1-c)^N}-1}\frac{\ln(1-(1-c)^N)}{(1-c)^N} \left(\frac{(1-(1-c)^N)\ln(1-(1-c)^N)}{(1-c)^N}+2\right) \tag{20}$$

Step (d). Substituting $x=(1-c)^N$ into inequalities (19), we have $-1 < \frac{(1-(1-c)^N)\ln(1-(1-c)^N)}{(1-c)^N} < 0$, implying $1 < \left(\frac{(1-(1-c)^N)\ln(1-(1-c)^N)}{(1-c)^N}+2\right) < 2$. Therefore, the following inequality holds,

$$\frac{\ln(1-(1-c)^N)}{(1-c)^N} > \frac{\ln(1-(1-c)^N)}{(1-c)^N}\left(\frac{(1-(1-c)^N)\ln(1-(1-c)^N)}{(1-c)^N}+2\right).$$

That is, the following inequality would imply that inequality (20) holds,
$-\frac{3(2N+1)}{N} > (1-(1-c)^N)^{\frac{1}{(1-c)^N}-1}\frac{\ln(1-(1-c)^N)}{(1-c)^N}$
By $(1-(1-c)^N)^{\frac{1}{(1-c)^N}-1} \in (e^{-1},1)$ and $(1-c)^N<1$, we have

$$\begin{aligned}(1-(1-c)^N)^{\frac{1}{(1-c)^N}-1}\frac{\ln(1-(1-c)^N)}{(1-c)^N} &< e^{-1}\frac{\ln(1-(1-c)^N)}{(1-c)^N}\\ &< e^{-1}\ln(1-(1-c)^N).\end{aligned}$$

Therefore, to prove inequality (20) we need to establish the following inequality,

$$-\frac{3(2N+1)}{N} > e^{-1}\ln(1-(1-c)^N) \tag{21}$$

Step (e). Solving inequality (21), we arrive at

$$c(N) =: 1-(1-e^{\frac{-3e(2N+1)}{N}})^{\frac{1}{N}}$$

It is easy to check that $0<c(N)<1$ for any given N. By our definition of $c(N)$, whenever $c\in(0,c(N))$, we have $\frac{\partial^2 g(N,c)}{\partial c^2} < \frac{\partial^2 f(N,c)}{\partial c^2}$, which implies $g(N,c)<f(N,c)$. Going back to the case of $N=4$, we have $c(4)=1-(1-e^{\frac{-27e}{4}})^{\frac{1}{4}} > 2.6\times10^{-9}$. That is, for $c\in(0,2.6\times10^{-9})$, we have $f(N,c)>g(N,c)$ for $N=4$. Now, as long as we can show that the iteration process indicated by the difference equation (Equation (15)) can reach a t such that $c_t^4 < 2.6\times10^{-9}$, we would have shown that the infima of the sequence $\{c_t^4\}$ is zero. Indeed, with the help of MATLAB, we can calculate that such a t exists ($t=6,246,758$) such that $c^4_{(6246758)} < 2.6\times10^{-9}$.

Step 5. Compare the two sequences $\{c_t^N\}$, and $\{c_t^{N+1}\}$. Show that $c_t^N > c_t^{N+1}$, $\forall t\geq 1$.

First, we show that if $g(N, c_t^N) \geq g(N+1, c_t^{N+1})$, then $g(N, c_{t+1}^N) > g(N+1, c_{t+1}^{N+1})$. By Equation (16) and our definition for $g(N, c)$, $h(N, x)$, we get

$$\begin{aligned} g(N, c_{t+1}(c_t, N)) &= \left(1 - (1 - c_{t+1}(N, c_t))^N\right)^{\frac{1}{(1-c_{t+1}(N,c_t))^N}} \\ &= \left(1 - (1 - h(N, g(N, c_t)))^N\right)^{\frac{1}{(1-h(N,g(N,c_t)))^N}} \\ &= g(N, h(N, g(N, c_t))) \end{aligned}$$

Treating $g(N, c_t) = x \in (0, \frac{1}{e}]$ as a constant and consider the function

$$g(N, h(N, x)) = (1 - (1 - h(N, x))^N)^{\frac{1}{(1-h(N,x))^N}}$$

where $x \in (0, \frac{1}{e}]$. We claim that $\frac{\mathrm{d}g(N,h(N,x))}{\mathrm{d}N} < 0$.

$$\begin{aligned} \frac{\mathrm{d}g(N, h(N, x))}{\mathrm{d}N} &= \frac{\partial g}{\partial c}\frac{\mathrm{d}c}{\mathrm{d}N} + \frac{\partial g}{\partial N}\bigg|_{c=\frac{x}{N-(2N+1)x}} \\ &= (1 - (1 - c)^N)^{\frac{1}{(1-c)^N}}\left(\frac{1}{1 - (1 - c)^N} + \frac{\ln(1 - (1 - c)^N)}{(1 - c)^N}\right) \\ &\quad \left(-\frac{Nx(1 - 2x)}{(1 - c)(N - (2N + 1)x)^2} - \ln(1 - c)\right)\bigg|_{c=\frac{x}{N-(2N+1)x}} \end{aligned}$$

Using inequality (19), we obtain $\frac{1}{1-(1-c)^N} + \frac{\ln(1-(1-c)^N)}{(1-c)^N} > 0$, so the sign of $\frac{\mathrm{d}g(N,h(N,x))}{\mathrm{d}N}$ is the same as that of $-\frac{Nx(1-2x)}{(N-(2N+1)x)^2} - (1-c)\ln(1-c)$. Using inequality (19) again, we have $-(1-c)\ln(1-c) < c$. So if $-\frac{Nx(1-2x)}{(N-(2N+1)x)^2} + c < 0$, then $\frac{\mathrm{d}g(N,h(N,x))}{\mathrm{d}N} < 0$. Since $c = \frac{x}{N-(2N+1)x}$, inequality $-\frac{Nx(1-2x)}{(N-(2N+1)x)^2} + c < 0$ is equivalent to inequality $-\frac{N(1-2x)}{(N-(2N+1)x)} + 1 < 0$. Since $x \in (0, \frac{1}{e}]$, we have $-\frac{N(1-2x)}{(N-(2N+1)x)} + 1 = -\frac{x}{(N-(2N+1)x)} < 0$. So we get $\frac{\mathrm{d}g(N,h(N,x))}{\mathrm{d}N} < 0$. It means that if we have two points c_t^N and c_t^{N+1} in sequences $\{c_t^N\}$ and $\{c_t^{N+1}\}$ respectively, s.t. $g(N, c_t^N) = g(N+1, c_t^{N+1}) = x \in (0, \frac{1}{e}]$ (e.g., $g(N, c_0^N) = g(N+1, c_0^{N+1}) = \frac{1}{e}$), we must have $g(N, c_{t+1}^N) > g(N+1, c_{t+1}^{N+1})$. Further more, if we have $g(N, c_t^N) > g(N+1, c_t^{N+1})$,we also have $g(N, c_{t+1}^N) > g(N+1, c_{t+1}^{N+1})$. To see this, since $\frac{\partial g(N,c)}{\partial N} > 0$, we have $g(N+1, c_t^N) > g(N, c_t^N)$. By continuity of function $g(N, c)$, we know there must exist a $\widetilde{c_t}^{N+1} \in (c_t^{N+1}, c_t^N)$ s.t. $g(N, c_t^N) = g(N+1, \widetilde{c_t}^{N+1})$, which implies $g(N, c_{t+1}^N) > g(N+1, \widetilde{c_{t+1}}^{N+1})$. By $\frac{\partial c_{t+1}(N,c_t)}{\partial c_t} > 0$ we know $c_{t+1}(N+1, \widetilde{c_t}^{N+1}) > c_{t+1}(N+1, c_t^{N+1})$, i.e. $\widetilde{c_{t+1}}^{N+1} > c_{t+1}^{N+1}$, which implies $g(N+1, \widetilde{c_{t+1}}^{N+1}) > g(N+1, c_{t+1}^{N+1})$ (as function $g(N, c)$ monotonically increases in c). Taken together, we have $g(N, c_{t+1}^N) > g(N+1, c_{t+1}^{N+1})$.

Second, we show that $g(N, c_t^N) \geq g(N+1, c_t^{N+1})$ implies $c_{t+1}^N > c_{t+1}^{N+1}$. Re-write Equation (16) as $c_{t+1}(N, c_t) = h(N, g(N, c_t))$. Then, since $\frac{\partial h(N,c)}{\partial N} < 0$, $g(N, c_t^N) =$

$g(N+1, c_t^{N+1})$ implies $c_{t+1}^N > c_{t+1}^{N+1}$. Similarly, $g(N, c_t^N) > g(N+1, c_t^{N+1})$ also implies $c_{t+1}^N > c_{t+1}^{N+1}$. This is because, since $g(N+1, c_t^N) > g(N, c_t^N)$ (due to g monotonically increasing in N) and the fact that g is a continuous function, there must exist a $\widetilde{c_t}^{N+1} \in (c_t^{N+1}, c_t^N)$ s.t. $g(N, c_t^N) = g(N+1, \widetilde{c_t}^{N+1})$, which implies $c_{t+1}^N > \widetilde{c_{t+1}}^{N+1}$ as we just proved. Further, as $\frac{\partial c_{t+1}(N, c_t)}{\partial c_t} > 0$, we know $\widetilde{c_{t+1}}^{N+1} > c_{t+1}^{N+1}$. That is, $c_{t+1}^N > c_{t+1}^{N+1}$.

The previous two steps have established that $g(N, c_t^N) \geq g(N+1, c_t^{N+1})$ implies both $g(N, c_{t+1}^N) > g(N+1, c_{t+1}^{N+1})$ and $c_{t+1}^N > c_{t+1}^{N+1}$. Now consider the sequences $\{c_t^N\}$ and $\{c_t^{N+1}\}$. Since $g(N, c_0^N) = g(N+1, c_0^{N+1}) = \frac{1}{e}$, we know $g(N, c_1^N) > g(N+1, c_1^{N+1})$ and $c_1^N > c_1^{N+1}$. Then $g(N, c_1^N) > g(N+1, c_1^{N+1})$ implies $g(N, c_2^N) > g(N+1, c_2^{N+1})$ and $c_2^N > c_2^{N+1}$. This iterative process proves $c_t^N > c_t^{N+1}, \forall t$.

Step 6. Show (13) hold for $N \geq 4$, $\forall c \in (0,1)$.

Step 5 established that $c_t^N > c_t^{N+1}, \forall t > 0$ and $\forall N > 0$, which implies that the infima of the sequences $\{c_t^N\}$ are non-increasing in N. Together with the fact that the infima of the sequence $\{c_t^4\}$ is zero (step 4) and $c_t^N > 0$ (step 3), the infima of the sequence $\{c_t^N\}$ (for any N) must be zero, which implies inequality (13) holds for $N \geq 4$, $\forall c \in (0,1)$.

Step 7. Show that $EHQ^{Sim}(n,c) > EHQ^{Seq}(n,c)$ holds for $n < 5$.

Recall that we only need to show that

$$(1-(1-c)^{n-1})^{\frac{1}{(1-c)^{n-1}}} \frac{(n-1)c}{(n-1)c + \frac{1}{1-c}} \leq EHQ^{Sim}(n,c)$$

(Equation (12)) holds for $n = 2, 3$ and 4. Again, we rewrite $EHQ^{Sim}(n,c)$ as $\frac{\frac{1}{1-c}+(n-1)c}{1+(n-1)c} \frac{nc}{2nc-c+1} \frac{(n-1)c}{(n-1)c+\frac{1}{1-c}}$. Since $\frac{\frac{1}{1-c}+(n-1)c}{1+(n-1)c} \geq 1$, we only need to show

$$(1-(1-c)^{n-1})^{\frac{1}{(1-c)^{n-1}}} < \frac{nc}{2nc-c+1}$$

Replacing $n-1$ with N, the above inequality becomes

$$(1-(1-c)^N)^{\frac{1}{(1-c)^N}} < \frac{(N+1)c}{2Nc+c+1}$$

Now, in Step 4. (steps (a) $\sim$ (e)), we have proved that for any N, there exists a $c(N)$, such that $\forall c \in (0, c(N))$, $(1-(1-c)^N)^{\frac{1}{(1-c)^N}} < \frac{Nc}{2Nc+c+1}$. This would imply that

$$(1-(1-c)^N)^{\frac{1}{(1-c)^N}} < \frac{(N+1)c}{2Nc+c+1}$$

Therefore, using the same logic as in Step 4., for the rest of the proof, we only need to check that the sequence induced by

$$\frac{(N+1)c_{t+1}}{2Nc_{t+1}+c_{t+1}+1}-(1-(1-c_t)^N)^{\frac{1}{(1-c_t)^N}}=0$$

can iterate from $c_0 = 1$ to $c(N)$, for $N = 1, 2$ and 3 (thus $n = 2, 3,$ and 4). It is easy to compute that $c(3) > 1.8 \times 10^{-9}$, $c(2) > 6 \times 10^{-10}$ and $c(1) > 2 \times 10^{-11}$. With the help of MATLAB, we can show $c^3_{(61)} < 1.8 \times 10^{-9}$, $c^2_{(42)} < 6 \times 10^{-10}$ and $c^1_{(31)} < 2 \times 10^{-11}$.

□

F Proof for Proposition 5

Proof. Let $\psi(a) = a - \frac{1-a^c}{ca^{c-1}}$. $\psi'(a) = 1 - \frac{1-a^c-c}{ca^c}$. $\psi'(a) > 0$ if and only if $a > \hat{a} = (\frac{1-c}{1+c})^{\frac{1}{c}}$. And $\psi(a) > 0$ if and only if $a > \tilde{a} = (\frac{1}{1+c})^{\frac{1}{c}}$. It is easy to see that $\tilde{a} > \hat{a}$.

As Myerson (1981) shows, the expected total effort can be expressed as

$$ETQ = \int_A \sum_{i \in N} \psi(a_i)p_i(a)\, \mathrm{d}F^N(a)$$

where $A = (0, 1)^N$.

Then whenever there exists at least one player with ability higher than $\tilde{a}$, the part $\sum_{i\in N}\psi(a_i)p_i(a)$ in a simultaneous contest will be higher than that in a sequential contest, since the prize will be allocated to the one with the highest ability. The probability that everyone's ability is lower than $\tilde{a}$ is $F^N(\tilde{a}) = (\frac{1}{1+c})^N \xrightarrow{N\to\infty} 0$, which implies our Proposition 5.

□

G Proof for Proposition 6

Proof. Let the utilization ratio in a contest of n players with an ability distribution parameter c be $U(n, c)$. We first derive the utilization ratio in simultaneous contests, $U^{Sim}(n, c)$. The total effort in a simultaneous contest is $ETQ^{Sim}(n, c) = \frac{n(n-1)c^2}{(nc+1)(1+(n-1)c)}$, hence $U^{Sim}(n, c) = \frac{2nc-c+1}{nc+1}$, which is always less than 2. Substituting n with 2, we have $U^{Sim}(2, c) = 2 - \frac{1+c}{1+2c}$.

$$\begin{aligned} ETQ^{Seq}(2,c) &= \tfrac{(1-c)c^{\frac{2-c}{1-c}}}{1+c-c^2} + \int_0^1 \int^1_{(a_1c)^{\frac{1}{1-c}}} (a_1c)^{\frac{1}{1-c}} da_2^c da_1^c \\ &= 2\tfrac{(1-c)c^{\frac{2-c}{1-c}}}{1+c-c^2} - \tfrac{(1-c)c^{\frac{2}{1-c}}}{1+2c-c^2}. \end{aligned}$$

And $U^{Seq}(2, c) = 2 - c^{\frac{c}{1-c}}\frac{1+c-c^2}{1+2c-c^2}$, which is also less than 2. The fact that $U^{Sim}(2, c) < U^{Seq}(2, c)$ is implied by the following inequalities $\frac{1+c}{1+2c} > \frac{1+c-c^2}{1+2c-c^2} > \frac{1+c-c^2}{1+2c-c^2}c^{\frac{c}{1-c}}$.

□

H Experimental Instructions: SEQ-G2

Name: ID Number: Total Payoff:

This is an experiment in decision-making. You will make a series of decisions in the experiment, followed by a post-experiment questionnaire. **Please note that you are not being deceived and everything you are told in the experiment is true.**

Each of you has been assigned an experiment ID, i.e. the number on your index card. The experimenter will use this ID to pay you at the end of the experiment.

Rounds: The experiment consists of 20 rounds of three-person games. The first 2 rounds are practice rounds, i.e., you will not receive payments from these two rounds. The payment you earned in each of the remaining 18 rounds will be cumulated toward your final payment.

Grouping: At the beginning of each round, you will be randomly grouped with two other people in the room. You are equally likely to be grouped with anyone in the room.

Endowment and Prize: At the beginning of each round, each of you will be given 120 tokens. You will use these tokens to compete for a prize which is worth 100 tokens.

Winning: In each round of the game, you will choose an effort level and the person with the highest effort level among the three will receive the prize of 100 tokens.

Procedure: The three participants in each group are called: Participant 1, Participant2 and Participant 3 based on the order in which they enter the game. The order is randomly assigned by the computer at the beginning of each round.

1. If you are Participant 1, you will be the first one to choose your effort. After observing your effort, Participant 2 and 3will choose their efforts subsequently.

2. If you are Participant 2, you will first observe Participant 1' effort and then choose yours. Both yours and the Participant 1's efforts will be observed by Participant 3 before he/she makes a decision.

3. If you are Participant 3, you will observe Participant 1's and 2's efforts first and then choose yours.

Effort Range: Your effort can be any number with four decimal points between [0, 120]. Tie-Breaking: If two or three of you make the same highest effort, the computer will randomly choose one as the winner.

Ability Factor: The ability factor captures the idea that it sometimes costs more or less to make the same effort: A person with a higher ability has a lower cost. At the beginning of each round, the computer will randomly draw a different ability factor for everyone, following the distribution described below. Your ability factor is private information: only you know it and do not inform any other participant of your private information.

Your ability factor is a number randomly drawn between $[0, 1]$, according to the following distribution function: $F(x) = x^{0.25}$. Do not worry if you do not understand this equation the

Table 10: Ability Distribution

Ability Factor	**Percentile**
0	0
0.004	25%
0.06	50%
0.32	75%
1	100%

distribution will be illustrated in a table and a graph below. The table below shows the percentiles of the distribution of ability factors.

For instance, the 25th percentile is 0.004. It indicates that with a 25% chance, your ability factor is below 0.004. Thus with a 75% chance, it is above 0.004.

The 50th percentile is 0.06. It indicates that with a 50% chance, your ability factor is below 0.06 whereas with a 50% chance, it is above 0.06.

For your reference, we also provide you with a graph of the distribution of the ability factor below. As you can observe in the graph, you are more likely to receive a low ability factor than a high ability factor.

[Insert Figure]

Your net earnings in a game: In each round, your earnings in the game will be determined by (1) your effort; (2) other participants' efforts; (3) and your ability factor. Specifically,

Your net earnings in a game= The amount of the prize if you win-Your Cost = 100 or 0-your effort/ability factor For example, if in a given round, the computer draws an ability factor of 0.5 for you, and you choose an effort of 10, then

(1) If you win, then your earning will be 100-10/0.5 = 80 tokens

(2) If you lose, then your earning will be 0-10/0.5 = -20 tokens

Calculator: To help you calculate your earning, we provide a calculator on the top-right corner of your screen. If you enter an effort level that you are considering in the box and then click the "compute" button, your cost, net earnings if you win, net earnings if you lose will be shown in the result box. Remember, you will always pay your cost, no matter whether you win or lose. Moreover, if the calculator shows that your net earnings are negative even when you win, it suggests that the effort level you considered is too high. Predicting Your Winning Probability:

Before you choose an effort, you will be given an opportunity to earn extra money by predicting how likely you will win the game. You will be asked the following questions:

Given your ability factor: x, and your are Participant y, estimate the probability that you will win ()?

Estimate the probability that any other participant will win ()?

If you think there is a 90% chance that you will win and a 10% that someone else will win, answer 90 to the first question and 10 to the second. If you think there is a 67% chance that you will win and hence a 33% that someone else will, answer 67 to the first question and 33 to the second. Each number you answer should be an integer between 0 and 100 and the two numbers have to add up to 100.

You are paid based on the accuracy of your prediction. The more accurate your belief is, the more you will earn. Since your prediction is made before you know others' efforts, the best thing you can do is to simply state your true belief.

If you believe that you will win with a 100% chance and you actually win, you will be rewarded 2 tokens. If you believe that someone else will win with a 100% chance (in other words, you will lose for sure) and someone else actually wins, you will be rewarded 2 tokens as well.

Here's another example. Suppose you believe that you will win with a 90% chance and someone else will win with a 10% chance.

1. If you win, your prediction payoff is:

$$2 - (1 - 90\%)^2 - (0 - 10\%)^2 = 1.98$$

2. If someone else wins, your prediction payoff is:

$$2 - (0 - 90\%)^2 - (1 - 10\%)^2 = 0.38$$

Your Total Payoff in Each Round:

Your Payoff in each round= Your net earnings in the game +Your endowment of 120 tokens + Your payoff in predicting your winning probability

Cumulative Payoff: Your cumulative payoff will be the sum of your payoff in all paying rounds.

Feedback: At the end of each round, you will receive the feedback on your screen about the round.

History: Your ability factor, all three participants' efforts, your Participant ID in the group (1/2/3), and your net earnings in the game in each previous round will be displayed in a history box.

Exchange Rate: At the end of the experiment, the tokens you earned will be converted to U.S. dollars at the rate of $1 = 110$ tokens.

Please do not communicate with one another during the experiment or use your cell phone. No food is allowed in the lab either. If you have a question, feel free to raise your hand, and an experimenter will come to help you.

I Post-Experiment Questionnaire

We are interested in whether there is a correlation between participants' decision behavior and some socio-psychological factors. The following information will be very helpful for our research. This information will be strictly confidential.

1. Gender
   (a) Male
   (b) Female
2. Ethnic Background
   (a) White
   (b) Asian / Asian American
   (c) African American
   (d) Hispanic
   (e) Native American
   (f) Other
3. Age
4. How many siblings do you have?
5. Grad/Year
   (a) Freshman
   (b) Sophomore
   (c) Junior
   (d) Senior
   (e) $>$ 4 years
   (f) Graduate student
6. Major
7. Would you describe yourself as (Please choose one)
   (a) Optimistic
   (b) Pessimistic
   (c) Neither
8. Which of the following emotions did you experience during the experiment? (You may choose any number of them.)

(a) Anger
(b) Anxiety
(c) Confusion
(d) Contentment
(e) Fatigue
(f) Happiness
(g) Irritation
(h) Mood swings
(i) Withdrawal

9. In general, do you see yourself as someone who is willing, even eager, to take risks, or as someone who avoids risks whenever possible? [7 point likert]

10. Concerning just personal finance decisions, do you see yourself as someone who is willing, even eager, to take risks, or as someone who avoids risks whenever possible? [7 point likert]

11. In general, do you see yourself as someone who, when faced with an uncertain situation, worries a lot about possible losses, or someone who seldom worries about them? [7 point likert]

12. Concerning just personal finance decisions, are you someone who, when faced with an uncertain situation, worries a lot about possible losses, or someone who seldom worries about them? [7 point likert]

13. In general, how competitive do you think you are? [7 point likert]

14. Concerning just sports and leisure activities, how competitive do you think you are? [7 point likert]