Collusion, Coordination and Delegation under Multitask

Abstract: In this paper, we will analyze the coordination and its effect on delegations under multitasks. In an organization, there are three participants, principal, supervisor and agent, with two kinds of tasks. The task one is decision-making of production that the principal arranges production according to agent's type and supervisor's information about agent's type, with collusion between the supervisor and agent. The task two is decision-making of investment that the principal decides whether or not develops new products correlating with old product according to supervisor's information about potential profitability of new product, involving cheap-talking between principal and supervisor. As for decision-making of investment, there exists interest conflict between principal and supervisor, but the interest conflict is contingent, which we define as the interest relation in an organization. Because supervisor's information about production and investment is correlated, supervisor will has more strategic considering as he reports his information to the principal, or chooses his action in the two tasks. Therefore, supervisor must coordinate the manager's information utilizing in the different tasks in order to maximize his whole profit, that is, the trade-off between the two tasks, or the interaction of the two incentives. Due to the two different tasks, there are two kinds of delegation for supervisor, with the first delegation in production and the second delegation in investment. We also study when delegation reach the goal of coordination of the two tasks in equilibrium. Our main findings is following: 1) the coordination between the two tasks is mainly determined by the interest relation, also influenced by interest conflict between the principal and supervisor in investment and supervisor's knowledge about production, and coordination is complementary under coinciding interest relation while substitute under conflicting interest relation; 2)the first delegation can reach the goal of coordination under any interest relation for it only grants supervisor partial authority in production under complete contract, but it just reach the aim of coordination under coinciding interest relation when the principal lacks commitment; 3)the second delegation can only possible reach the aim of coordination under conflicting interest relation for it grants the supervisor full authority in investment.

Key Words: Interest relations coordination delegation
Interest conflict knowledge

Collusion, Coordination and Delegation under Multitask

1. Introduction

When shareholder or superior is facing with the choice between the two Cates, “fish” and “bearcat” in an organization, how will trade off between these two? Could he get both Cates at the same time? If he could not get these two, when he chooses “fish” at the cost of “bearcat”, or otherwise? Does the choice between “fish” and “bearcat” have any effect on delegation in an organization, or in other words, can shareholder reach the goal of trade-off between these two through delegation? These two problems, coordination of delegation under multitask are the central of this paper.

The coordination of different tasks is prevalent phenomenon in an organization and there are many different kinds of coordination. One kinds of coordination in an organization is Milgrom and Holmstrom(1991) classic story about coordination between two efforts. The professor’s efforts for teaching and research, the taxi driver’s efforts for driving and maintaining the car, and marketing manager’s effort for market and service for consumers, are belong to this kind coordination. Another kind coordination in an organization is trade-off among the use of scare resource under multitask, such as example in Hart and Moore(1999). The fund for advertising or producing in a firm, the resources for new market or old market, and the Union donating developing countries directly, or developing their education. Maybe more interesting coordination among different tasks is the balance of utilizing the information in different tasks when the information for different tasks is correlated.

One of main contribution in this paper is the first step in our own knowledge to deal with the coordination of information utilizing under multitask with cheap talking and collusion. As the information made use of in one task is correlated with that in the other, so there exists the trade-off between utilizing the information in different tasks. The interest relations between principal (shareholder) and supervisor (manager)

determine the desire for supervisor to collude in the task of production as principal use supervisor's information in it as well as the cost use supervisor's information in the task of investment, thus it is the main force to decide the coordination of the two tasks. So compared with solo task increase, principal can make more effective use of the supervisor's information in both tasks and increase his profit in both tasks when interest relation is coinciding, that is, the coordination of the two tasks is complementary; meanwhile principal can only make more effective use of information in one task at the expense of the other under conflicting interest relation, that is, the coordination of the two tasks is substitute. Since the interest conflict decides the cost for principal to utilizing supervisor's information in investment as well as the supervisor's knowledge about production decides the value to utilizing the supervisor's information in production, thus these two factors determine when principal utilize the supervisor's information in task of production at the cost of investment.

The second main contribution is we analyze the effect of coordination on delegation under multitask; namely, when the delegation can the goal of coordination under multitask. The first delegation[1] grants part authority for supervisor in the task of production, and principal still control the use of supervisor information in production as well as the desire for supervisor to collude by the first delegation, thus it can always reach the goal of coordination when principal has full commitment. However, the first delegation can not always control the desire for supervisor to collude and reach the aim of coordination if principal lacks commitment in investment. It can arrive at the goal of coordination only under coinciding interest relation. Since the second delegation grants full authority for supervisor in the task of investment, it cost principal at least not to utilize supervisor's information in investment, which can only reach the goal of coordination in equilibrium when the coordination between the two tasks is fully substitute. So it could only occur under conflicting interest relation, but for it emerges in equilibrium, the second delegation must result in that the increase of principal's profit is high enough to make up for his loss in investment from second delegation.

[1] The first delegation in this paper means that principal grants supervisor part authority in production and supervisor becomes sub-principal of agent which is the same to delegation in Laffont and Martimort(1998), while the second delegation gives supervisor full authority in investment which is the same to delegation in Dessein(2003).

Because the second delegation is a committing mechanism, so it could more like to reach the goal of coordination when principal is short of commitment than when he has full commitment.

Thirdly, we also shed some light on authority allocation under multitask and relationship between delegation and interest relation under multitask. Because different interest relation results in different coordination, so the different delegations occur in different interest relations, which means different interest relations are corresponding to different authority in an organization. For the same reason, the first delegation is easier to occur under coinciding interest relation than it under conflicting interest relation. More interesting, we find the second delegation can just occur under conflicting due to the coordination of the two tasks, and with interest conflict between principal and supervisor in the task of investment increasing, the second delegation that grants supervisor full authority is more like to occur.

As for coordination in an organization, Holmstrom and Milgrom(1991) argue that fixed wage is optimal for coordinating the different incentives in different tasks; Holmstrom and Milgrom(1994)discuss the interaction of multi-incentives tools under single task. As for the first delegation and collusion, Laffont, Martimort(1998) manifest that decentralization dominates centralization when there collusion and difficulty in communication exist because decentralization redesigns bargaining power in an organization; Grimaud, Laffont and Martimont (2003) prove the equivalence principle about organization design, that is, the allocation in centralization is equivalence to allocation in decentralization under collusion between agent and supervisor, while Baliga and Sjostrom (1998)gain similar conclusion in moral hazard setting. As for second delegation and cheap talking, Dessein(2003) argues that delegation is other means of communication, and it will takes the place of direct communication when interest conflict is small, based on the seminar work of Crawford and Sobel’s; Li and Wing (2003) analyzed the delegation to one expert, or two experts under similar approach. As for coordination and delegation, Hart and Moore (1999) argue that the authority design in an organization is the means to coordinate to make use resources in this organization under incomplete when there is conflict among different use of these

resources; Athey and Roberts(2002)analyze that interaction of incentive and authority with multiple tasks.

2. The model

2.1. A Heuristic Sketch

Before getting into the details of the model, a verbal summary of our problem and its mechanism may be useful. Consider an organization with three participants, shareholder, manager and worker, and two kinds of tasks. The task one is decision-making of production that shareholder arranges production in the organization according to agent's type and manager's information about agent's type, with collusion between manager and agent. The task two is decision-making of investment that shareholder decides whether or not develops new products correlating with old product according to manager's information about potential profitability of new product, involving cheap-talking between shareholder and manager. As for decision-making of investment, there exists interest conflict between shareholder and manager, but the interest conflict is contingent, which we define as the interest relation in an organization. Because manager's information about production and investment is correlated, manager will has more strategic considering as he reports his information to shareholder, or chooses his action in investment. Therefore, shareholder must coordinate the manager's information utilizing in the different tasks in order to maximize his whole profit, that is, the trade-off between the two tasks, or the interaction of the two incentives.

As for coordination, the interest relation between shareholder and manager is the determining factor. As in the task of production, manager always has incentive to manipulate information about production in order to get information rent from worker, so there are always conflict between shareholder and manager in decision-making of production. However, the desire for manager to manipulate information in production is decided by the interest relation because the manipulation in production will affect his benefit in investment with supervisor's information in production correlating with it in investment. As in the task of investment, the desire for manager to manipulate information about investment is also determined by the interest relation. Therefore,

shareholder is able to utilize the manager's information in both tasks, that is, the coordination of the two tasks is complementary under coinciding interest relation while shareholder can utilize the manager's information only in one task, that is, the coordination is substitute.

As for delegation, the first delegation grants manager part authority in the task of production and becomes sub-principal of worker, under which shareholder can still control the desire for manager to collude with agent; meanwhile the second delegation grants manager full authority in investment with manager making decision in investment, which cost shareholder at least some loss in investment. As shareholder still control information utilizing in production under it, the first delegation can always reach the goal of coordination if shareholder can make credible in investment, but it can just reach the goal of coordination if shareholder lacks commitment in investment. As for the second delegation costs shareholder much, it can only reach the aim of coordination under conflicting interest relation, moreover, it can be only means to coordinate the two tasks when shareholder lacks commitment in investment for it is committing mechanism. To sum up, we have:

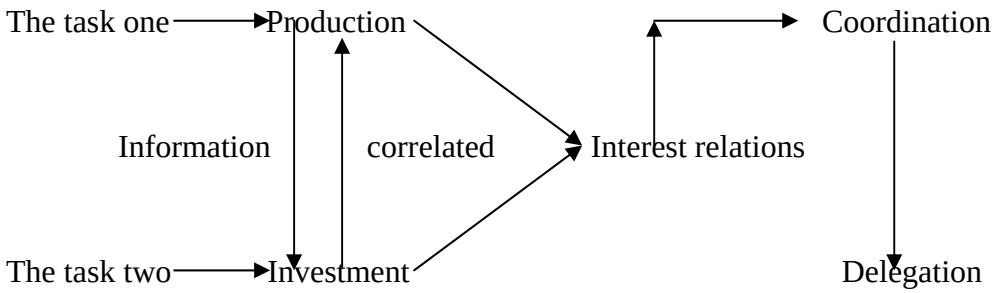

Figure1: coordination and delegation under multitask

2.2. The Model in Detail

• ***Participants and tasks:*** In an organization there are three participants, principal, supervisor and agent, and all of them are risk neutral; at the same time two tasks, production and investment. Due to being the owner of the vertical structure, the principal is the formal authority in an organization with the right of making all the

decisions initially.

As for the task of production, the agent is the productive unit, and principal arranges production according to the production parameter and supervisor's information about production. The production x created by agent's activity depends on a production parameter θ, which is also called as agent's type, and the agent's e, that is

$$x = \theta + e$$

The agent's disutility of effort is equal, in money terms, to $e^2/2$, and principal receives the production x and gives wage W to the agent. So the agent's utility function is

$$U(W, e) = W - e^2/2$$

There exists an ex-ante competitive supply of agents, with reservation utility U_0, which is normalized to zero. The agent's participating constraint is

$$EU(W, e) = E(W - e^2/2) \geq 0$$

The agent only involves directly in the task of the production while the supervisor involves the two. The supervisor not only knows the investment parameter, but also knows something about the production parameter (it will describe in detail in information assumption). The supervisor exerts on effort, and receive wage S from principal, also with reservation utility zero, so his participation constraint is

$$ES \geq 0$$ [2]

However, he is constrained by limited liability, that is $S \geq 0$.

As for the task of investment, it is directly connected with the principal and the supervisor, which means investment activities bring about the principal and the supervisor benefit (or loss). However the amount of benefit depends both on the investment parameter t_j , or the supervisor's type and action taken in investment. The following table indicates the benefit from investment for the principal and the supervisor

[2]If we consider the benefit(loss) from investment in participating constrain, then the loss or benefit from investment matters to coordination of the two incentives; moreover, the principal may be benefit from the increase in interest conflict as principal can transfer supervisor's benefit in investment. So when supervisor's investment benefit is considered in participant constrain, the interest conflict will have other effect and we ignore it in main body. As in discussion, we find our main conclusions are not greatly changed, even if we take account into investment benefit.

t_1 t_2

Actions / Participants	a_1	a_2
Principal	k	0
Supervisor	0	k_1

Actions / Participants	a_1	a_2
Principal	0	k
Supervisor	0	k_2

(2)

In this table, $k=(\Delta\theta)^2$, $k_1=k\cdot\alpha$, $k_2=k\cdot\beta$, and $\alpha>0$, $\beta=1$ or $\beta=-1$, α,β,k are common knowledge. This table is a simple description of the standard game of cheap talking. In different state the principal will have different favorable actions. In state t_1, the conflict of interest makes the principal and the supervisor prefer different actions. In state t_2, their interests can agree or against with each other. α represents the seriousness of the conflicts: bigger α is, severer the conflicts are; smaller α is, softer conflicts will become. β shows whether there are the common interest between principal and supervisor: when $\beta=1$, the principal and the supervisor have the common interest; when $\beta=-1$, their interest are quite opposite and have not any common interest.

(α,β) reflects the congruence interest and dissonance interest in different states, we define it as the interest relation between principal and supervisor, or the interest relation in an organization. When $\alpha<1,\beta=1$, the common interests outweigh the conflict interests, then we call it coinciding interest relation; when $\alpha>1,\beta=-1$, the interests between the principal and the supervisor are completely opposite, and the interest conflicts are quite severe, thus we call it conflicting interest relation; when $\alpha<1,\beta=-1$, their interests are against each other but the conflicts are not severe, and when $\alpha>1,\beta=1$, it has common interests but the conflicts are severe; then we call them middling interest relation. In fact, the interest relations described in this paper are the same to interest conflict in literature except they are contingent.

• ***Information:*** The production parameter(agent's type) can take two values: θ_1 and θ_2, such that $0<\theta_2<\theta_1$, and with the prior probabilities 1/2respectively. θ_1 and

θ_2 represents good(high) state of productivity and bad(low)productivity. Let $\Delta\theta = \theta_1 - \theta_2$.

The supervisor is uninformed about agent's type in production, but he receives a signal ω about the agent's type. τ is drawn from a discrete distribution $W = \{\omega_1 \ \omega_2\}$ with prior probabilities 1/2 respectively. The signal is observed both supervisor and agent. Hence, the information sets in production are nested along the hierarchy: nature reveals to the agent both his type and supervisor's information about production; only the latter is available to the supervisor while principal observes none of information[3]. The joint probabilities on $(\theta_i \ \omega_j)$ are defined as $p_{ij} = p(\theta = \theta_i, \omega = \omega_j)$, with $p_{ij} > 0$ for all ij. The correlation between the agent's type and the supervisor's signal about production is $p(\theta_1 \mid \omega_1) = p(\theta_2 \mid \omega_2) = q$, with $1/2 < q < 1$.[4]

The investment parameter (supervisor's type) can take two values: τ_1 and τ_2 with prior probabilities 1/2 too, which is known by the supervisor. However, the signal ω and supervisor's type τ is correlated, that is

$$\rho = p(\omega_1 \mid \tau_1) = p(\omega_2 \mid \tau_2) \ \text{ with } \ 1/2 < \rho \le 1$$

Though the assumption that the supervisor receive different signal about production and investment is more realistic, but it make analysis extremely complicated for collusion constrains between supervisor and agent become too much. So we further assume $\rho = 1$, then we have

$$p_{ij} = p(\theta = \theta_i, \tau = \tau_j) \ \text{ and } \ q = p(\theta_1 \mid \tau_1) = p(\theta_2 \mid \tau_2) \ \text{ with } \ 1/2 < q \le 1$$

With this information structure, we conjecture that main results in the paper have not change because the mechanism for trade-off between utilizing supervisor's information in the two tasks has not yet changed under this information assumption. Now, the information sets in the two tasks are nested along the hierarch again.

Because we have in fact assumed that correlation between the supervisor's type

[3] Nested information structures are standard in both the literatures on collusion and on delegation in hierarchies, and the information structures are the same as they in Grimaud, Laffont and Martimort(2003). The only role for this assumption is to simplify analysis. At the end of this paper, the nested information structures are replaced by the asymmetric one and it is found that all the conclusions make no changes as long as the collusion system keeps unchanged.

[4] To simplify analysis, q is restricted to the condition: $q \times (2 + \Delta\theta) < 2 - \Delta\theta$ and $2 - 3\Delta\theta > 0$ .The assumption guarantees that agent's effort in all states is positive and always gains information rent in the state of θ_1, so that we need not consider its participant constraint.

and the supervisor's information is invariable, so the correlation between the supervisor's type and agent's type q, plays the same role as it in Grimaud, Laffont and Martimort (2003). It represents the precision of supervisory information and actually reflects the supervisor's knowledge about the production --- the larger q is, the more information the supervisor has for supervisor and the more possible it is for him to figure out the exact production parameter; and vise versa. Hereby, q stands for the supervisor's knowledge about production.[5]

Lastly, we assume the agent's is not observed by the principal and the supervisor.

• ***The forming of coalition:*** Following the literature(Tirole 1986, Laffont and Martimort 1997, 1998, 2000), we assume that that the supervisor and the agent will collude in order to maximize their total utility.. As for the collusion between supervisor and agent, we assume:

(1) The side contract between the supervisor and the agent can be self-enforced;

(2) The side contract between supervisor and the agent is optimal for these two parties;

(3) The budget in the side contract is balanced under any state.

As for the detail about collusion mechanism, we follow Grimaud, Laffont and Martimort (2003), assuming supervisor has full bargaining power during collusion, which means that the supervisor design side contract between him and agent to maximize his own utility under multitask with the constrain of the agent participating and truth-telling. This assumption is just for simplifying analysis, not vital assumption. As a matter of fact, even if the-third-party mechanism in the collusion (Laffont and Mortimort, 1997, 1998, 2000) is employed, the mains conclusion is still correct, given the bargaining power of the supervisor is not zero.

• ***Delegation and authority allocation:*** Due to two different kinds of tasks, there are two different kinds delegation associating with those tasks. In the process of production decision, the supervisor is granted for partial authority and supervisor becomes sub-principal for agent in production when delegation occurs. Under the delegation in

[5]If the information structures are asymmetric, that is, agent only receives information about his own type, and then q also stands for agent's knowledge about investment. At the same time, if the correlation between supervisor's type and his signal about production is variable, then q in fact stands for the correlation between both tasks.

production, there are two noticeable differences with the case of centralization. The principal does not give any direct transfer, nor does he communicate directly with agent at the bottom of hierarchy. The right to contract with agent for the production is instead relinquished to the supervisor; at the same time, the principal only contracts directly with the supervisor for production. Second, the supervisor is provided with all the bargaining power in contracting with the agent. We call this kind of delegation first kind of delegation, and if it emerges in an organization, the supervisor is granted for the supervising right in production. In the process of investment decision, the supervisor is granted for full authority and owns the right to decide his investment action a_i .when delegation occurs. We call this kind of delegation the second delegation in an organization, and if it emerges, the supervisor is granted for the direct decision-making right in investment.[6]

The first delegation in this paper is the same as delegation in literature of collusion and organization design such as Baliga and Sjostrom (1998), Laffont and Martimort(1998) and Grimaud, Laffont and Martimort (2003). The second dlegation in this paper is the same as delegation in literature of cheap talking and organization design, for example delegation in Dessein(2003) and Li and Wing(2003). Departure from the literature, we study the coordinating effect of the delegation, or namely, the delegation is brought by the coordination between the tasks.[7]

• ***Timing:*** There are five dates, $t = 0,1,2,3,4$. At the date 0, the principal designs and allocation authority in the organization; at date 1 nature draws the agent's type θ_i and the supervisor's type τ_j for $i, j \in \{1,2\}$, with the agent receiving both, the supervisor only the type of his own and the principal none; at date 2, all kinds of contracts, whether grand contract or side contract, are signed by the principal ,the supervisor and the agent, given the authority structure in the organization; at date 3,the communication among the principal, the supervisor and the agent takes place on the condition of authority structure and the all kinds of signed contracts; at the date 4,

[6] In this paper, we do not the delegation for the agent because it is always weakly dominated by the corresponding centralization.

[7] In fact, when delegation plays the role as the means for coordination, it must be taken into account the sequential arrangement of the two tasks, especially under incomplete contract.

agent's effort is exerted and investment action is taken by the principal or the supervisor, then output and all kinds of contracts are enforced.

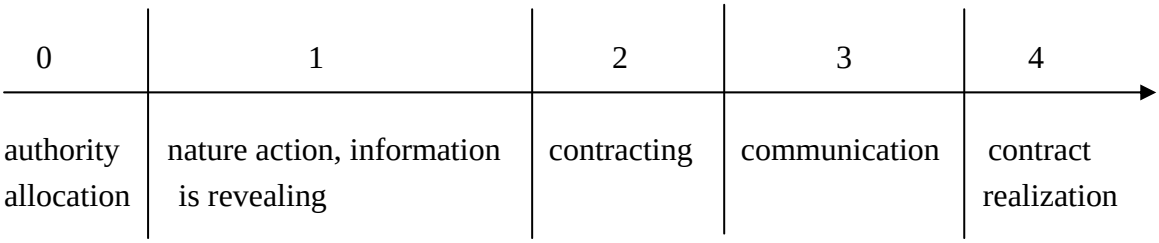

Figure 3: Timing

Here is the sum of the symbols and formulas used in this paper.

t : Sum of wages paid to the agent and the supervisor x : Output

W : The agent's wages S : the supervisor's wages

Z : Probability of taking action a_1 α : interest conflict

e : The agent's effort q : the supervisor's knowledge about production

$\pi_{NS} = \theta_2 + \frac{1}{2} + \frac{\Delta\theta^2}{2}$: The profit in production without supervisor's information

$\Delta r = \frac{2q-1}{(1-q)^2}\Delta\theta^2, \Delta r_1 = \frac{2q-1}{(1-q)q}\Delta\theta^2$: The temptation for the supervisor to collude

$\Delta M = \frac{(2q-1)^2}{4q(1-q)}\Delta\theta^2$: The maximizing increase of profit in production when supervisor's information about production is made use of

3. Collusion, Coordination and Delegation under Complete Contract

Now, we briefly discuss the coordination between the two tasks and its effect on the delegation under our multitask formulation[8]. The coordination between the two tasks is the trade-off between utilizing the supervisor's information in the tasks of production and utilizing the supervisor's information in the tasks of investment, as they are correlated.[9] However, utilizing the supervisor's in production is determined by the

[8] The discussion about coordination and delegation is still applied in incomplete contract, and we ignore it under incomplete contract.

[9] In this paper, utilizing the supervisor in production is means that the principal controls the agent well, reflected by the difference in the agent effort in state in 22 and 21; utilizing the supervisor's information in investment means the principal gets his benefit in investment reflected in that the investment choice that principal prefers is implemented.

incentive for the supervisor to collude with the agent while it in investment is decided by the incentive for the supervisor to tell truth. Therefore, the coordination the two tasks is essentially the coordination between the incentive for supervisor to collude with agent in production and the incentive for supervisor to tell truth in investment, that is, the interaction of the two incentives. Furthermore, the utilizing information in production results in the profit increasing in it while utilizing information in investment also results in it, so the coordination is also reflected in trade-off between the increasing profit in production and the increasing profit in investment. So, we could confirm the interaction of the two incentives according to trade-off of the profits in the two tasks.

Definition1: The interaction of the two incentives is complementary if the principal's profit from any task more under multitask is more than it under single task; the interaction of the two incentives is substitute if the profit from one task are more than it under single task while it from the other is less; the interaction of the two incentives is independent if the profit from any task is the same as single task.

That interaction of the two incentives is complementary means that the incentives problem in the two tasks is coincident and the utilizing information in one task helps the information utilizing in the other in equilibrium; That it is substitute means that the incentives problem in the two tasks is conflicting and the utilizing information in one do against the utilizing information in the other;. That it is independent means that the incentives problem has nothing to do with each other.[10]

According to Grimaud, Laffont and Martimort (2003), Croworf and Sobel(1982) , the principal earns the profit of π_{NS} in production while $k/2$ in investment when make decision alone. Moreover, principal can at least gains a profit of $\pi_{NS}+k/2$ in the multi-task decision-making processes for he can at least gains $k/2$ in investment by always choosing the same action in investment, and then gains π_{NS} in production.[11] So when he just earns a profit of $\pi_{NS}+k/2$, then the principal make decision independently, and the two incentives have no interaction on each other.[12] If he gets

[10] The complement and substitute are the reflection of the trade-off between the two incentives, and the complement is the same as Milgrom and Roberts(1990,1995).

[11] In this paper, principal can always gain more profit under multi-task than aggregation of profit from single-task. Therefore, it is reasonable for principal to conduct the two decisions at the same time, or supervisor to assume two tasks, but in more general setting, it may be the profit from multi-task is less than the aggregation of single-task.

[12] In fact, that the two incentives are independent is also one kind of interaction of those incentives, but then the

more than it, the principal must have worked out a balance between the two incentives which are thus certain to have affected each other. All in all, it is necessary and sufficient for the two incentives to affect each other if the profit is more than $\pi_{NS} + k/2$.

Another concern in this paper is how coordination affects delegation, or in other words, when the delegation under multitask can reach the goal of coordination of the two incentives in equilibrium. When the profit is $\pi_{NS} + k/2$, the delegations in production and investment are the same as single task, so coordination can only influence delegation under multitask when the profit is more than $\pi_{NS} + k/2$.[13] Just for the incentive and delegation under multitasks are the same as in the single task when that the two incentives are independent, we focus on when the two incentives affect each other and its effect on delegation in the following. Since the profit is no more than $\pi_{NS} + k/2$ if the two kinds of delegation occur at same time，it is impossible for the coordination between the two incentives bring about the two delegations in equilibrium. To sum up, we have following conclusion:

Lemma1: If and only if the principal gets a profit more than $\pi_{NS} + k/2$, then the two incentives interact each other; the two delegation in equilibrium is never the result of the coordination under multitask.

3.1. Collusion-proofness principle

In a centralization organization, the principal directly contracts and communicates with both the supervisor and the agent. A grand-mechanism ruling the organization is quaternion a $GC = \{ W(m_s, m_a), S(m_s, m_a), x(m_s, m_a), z(m_s, m_a) \}$ stipulating monetary transfers respectively for agent and supervisor as well as output targets and investment choice as the supervisor's and the agent's messages (denote respectively m_s *and* m_a which belongs to two message spaces M_s and $\mathrm{M_a}$.

A centralized organization may be subject to coalition formation between the supervisor and the agent. The supervisor has all the bargaining power at the collusion contract. As the Revelation Principle applies at the side-contracting stage, and there is

---

delegation in an organization is the same as sole task. So we ignore this kind of coordination in an organization.

[13] According to Grimaud, Laffont and Martmort(2003) and Craworf and Sobel(1982), the delegation in production is result of the collusion and the delegation in investment is the result of the common interest when the interaction of the two incentives is independent.

no loss of generality in assuming that collusion-mechanism is a direct mechanism, that is, side contract is the solution of:[14]

$$\underset{\{\phi_\tau, y_\tau\}}{Max} P_{ij}\left[t(\phi_{ij}) - y_{ij} + \left[1 - z(\phi_{ij})\right]k_j\right]$$

$$s.t. \quad y_{1j} - \frac{\left[x(\phi_{1j}) - \theta_1\right]^2}{2} \geq y_{2j} - \frac{\left[x(\phi_{2j}) - \theta_1\right]^2}{2}$$

$$y_{1j} - \frac{\left[x(\phi_{1j}) - \theta_1\right]^2}{2} \geq \bar{U}_{1j}$$

$$y_{2j} - \frac{\left[x(\phi_{2j}) - \theta_2\right]^2}{2} \geq \bar{U}_{2j}$$

$\phi_\tau(\cdot)$ is a collective manipulation of the messages (m_s, m_a) sent to principal, which maps the agent's report to the supervisor into the set $\Delta(M_s, \mathrm{M}_a)$ of measures on the collective messages sets. $y_\tau(\cdot)$ stands for the total wage the agent gets in the collusion, including the sum of transfer payment from the supervisor and the principal. $t(\phi_{ij})$、$\mathrm{z}(\phi_{ij})$ and $x(\phi_{ij})$ means the sum of wage which the principal pays to the supervisor and the agent, investment choice and the planned output when the messages the supervisor reports to the principal is ϕ_{ij}.

Under the condition of complete contract, the Collusion-Proofness Principle still holds under multitask setting. The logic for it is similar to underlying the standard revelation principle: any equilibrium of the overall game of grand mechanism offer cum side contracting gives an allocation that can be replicated with direct grand mechanism G offered by the principal himself. Through the similar analysis like Grimaud, Laffont and Martimort(2003), we get the following conclusion.

Lemma2: A grand mechanism GC is collusion-proof if and only if such that the following coalition incentive constrains are satisfied:

$$t_{1j} - \frac{\left(x_{1j} - \theta_1\right)^2}{2} + \left[1 - z_{1j}\right]k_j \geq t_{ij} - \frac{\left(x_{ij} - \theta_1\right)^2}{2} + \left[1 - z_{ij}\right]k_j$$

[14] The incentive compatible constrain of θ_2 is not considered in the side contract, for, according to the general principle of the mechanism design, the incentive compatible constrain on the agent with lower abilities comes into satisfaction automatically, giving that of the agent with higher abilities does. This explanation can be proved right by the content in the appendices.

$$t_{2j}-\frac{\left(x_{2j}-\theta_2\right)^2}{2}+\left[1-z_{2j}\right]k_j-\frac{\lambda_j}{P_{2j}}\frac{\Delta\theta}{2}\left[2x_{2j}-\theta_2-\theta_1\right]$$

$$\geq t_{ij}-\frac{\left(x_{ij}-\theta_2\right)^2}{2}+\left[1-z_{ij}\right]k_j-\frac{\lambda_j}{P_{2j}}\frac{\Delta\theta}{2}\left[2x_{ij}-\theta_2-\theta_1\right]$$

$$P_{1j}=\lambda_j+u_j \qquad \lambda_j,u_j\geq 0, \qquad S_{ij}\geq 0 \qquad j=1,2,\ i=1,2$$

λ_j stands for the incentive-compatible Lagrange multiplier of the agent with type θ_1 and μ_j stands for the participating Lagrange multiplier with type θ_1. [15]

3.2. Coordination and delegation

On the base of the lemma 2, we use the approach similar to backward induction to analyze the coordination and delegation in an organization. First we figure out that centralization is optimal and allocation in the centralized organization; Then, we discuss the interaction of the two incentives; At last, we study the effect of coordination on delegation.

The same to cheap-talking, the communication between the principal and the supervisor, or the cost for utilizing information in investment is determined by the interest relations. However, the interest relations may also have something to the desire for the supervisor to collude because the collective manipulation of the messages in production influences the supervisor's benefit in investment. Given the interest relations, the interest conflicts determine the communication between the principal and the supervisor while the supervisor's knowledge determines the profit in production. Therefore intuitively, the coordination between the two incentives is decided by these three factors, and the following theorem justifies this intuition.

Theorem1: When $\alpha<1,\beta=1$, the interaction of the two incentives is complementary; when $\alpha>1,\beta=-1$ and if $2\Delta M'>k$, the interaction of the two incentives is substitute; the interaction of the two incentives is independent under any other condition. $\Delta M'$ is:

[15] Please see or refer to the analysis of Grimaud, Laffont and Martmort(2003) about the collusion-proof-principle. In the paper we haven't considered directly the supervisor's wealth bound. But taking it into account doesn't change the conclusion of lemma 2.

$$\Delta M' = \begin{cases} \frac{2q-1}{2} k_1 - \frac{q(1-q)}{4\Delta\theta^2} k_1^2 & k_1 \le \Delta r_1 \\ \Delta M & k_1 > \Delta r_1 \end{cases}$$

The theorem 1 confirms the intuition that the interaction of the two incentives is mainly determined by the interest relations, also affected by interest conflicts and supervisor's knowledge. The interest relation is the driving factor to determine the interaction of the two incentives, by not only deciding the desire for supervisor of truth-telling in investment, but also affecting the desire for supervisor to collude with agent in production. In making investment decision, the principal wants to communicate well with the supervisor and the desire for the supervisor to manipulate information in investment is determined by the interest relations between the supervisor and the principal. In making production decision, the principal always hopes to control the agent well with the supervisor's information about production. But the supervisor always hopes to manipulate the information in order to get the information rent from the agent. So there are always conflicts between the principal and the supervisor when making production decisions. Because the supervisor's information utilized in production and the supervisor's information utilized in investment are correlated, the manipulation in production must influence supervisor's benefit in investment, which is determined by the interest relations. Thus, the supervisor will balance what he will get from the two tasks according to the interest relations.

As the interest relations are given, then the interest conflicts are decisive factor for the communication between the principal and the supervisor in investment. In production, due to Grimaud, Laffont and Martimort(2003), the supervisor's knowledge is definitive factor to determine the profit in production. Thereby under the specific interest relations, the two factors also play a final important role to determine the interaction of the two incentives.

When the interest relations are coinciding, if principal's preferable actions are taken in investment, then

$$t_{11}-\frac{(x_{11}-\theta_1)^2}{2}\geq t_{12}-\frac{(x_{12}-\theta_1)^2}{2}+k_1$$
$$t_{12}-\frac{(x_{12}-\theta_1)^2}{2}+k\geq t_{11}-\frac{(x_{11}-\theta_1)^2}{2}$$

Compared with single production process, the desire for supervisor to collude with agent is increasing in state 11 while it is decreasing in state 12. Nevertheless, the compounded effect in state θ_1 is that the desire for collusion is decreasing because $k>k_1$ under this interest relations. So is it in state θ_2, and the incentive to collude is reduced. Compared with single investment process, the cost for principal to implement his preferable action, or utilize supervisor's information in investment is reduced because cost to induce supervisor to tell truth is also burdened by production process. So when the interest relations are coinciding, utilizing supervisor's information in production raises the desire for the supervisor to communicate with the principal in investment, and the same to utilizing supervisor's information in investment. In fact, we find when the interest relations are coinciding, the equilibrating allocation is

$$z_{ij}=(1,0,1,0)' \qquad \pi=\pi_{\mathrm{NS}}+\Delta(k_1)+k-k_1/2$$

$$\Delta(k_1)=\begin{cases}\dfrac{k_1(1-q)[(4q-2)\Delta\theta^2-(1-q)^2k_1]}{4q\Delta\theta^2} & \mathrm{k}_1\leq\Delta r\\ \Delta M & \mathrm{k}_1>\Delta r\end{cases}$$

$$e_{11}=e_{12}=1 \qquad \mathrm{e}_{21}=1-\frac{(1-q)}{\Delta\theta}\mathrm{k}_1 \qquad \mathrm{e}_{22}=1-\frac{(1-q)^2}{q\Delta\theta}k_1 \qquad \text{for } \mathrm{k}_1<\Delta r_1$$

$$e_{11}=e_{12}=1 \qquad \mathrm{e}_{21}=1-\frac{(1-q)}{q}\Delta\theta \qquad \mathrm{e}_{22}=1-\frac{q}{(1-q)}\Delta\theta \qquad \text{for } \mathrm{k}_1>\Delta r_1$$

The principal makes the profit increase in both tasks by balancing off the two incentives.

When the interest relations between the supervisor and the principal are conflicting, if the preferable actions are implemented, then

$$t_{11}-\frac{(x_{11}-\theta_1)^2}{2}\geq t_{12}-\frac{(x_{12}-\theta_1)^2}{2}+k_1$$
$$t_{12}-\frac{(x_{12}-\theta_1)^2}{2}-k\geq t_{11}-\frac{(x_{11}-\theta_1)^2}{2}$$

The desire for collusion in production is dramatically increasing while the cost for

utilizing information in investment is decreasing. However, the former effect dominates the latter. From above inequalities, we find that the incentive in production is collided with it in investment, or utilizing supervisor's information in production is contradicted with it in investment, which bring about the utilizing information in one task must be on the cost of the other. Hence the interaction of the two incentives must be substitute or independent. When interest conflicts are serious and the supervisor has much knowledge about production, the utilizing information in investment costs principal much while utilizing information in production benefits principal much, so it may be best choice for principal to utilizing the supervisor's information in production at the cost of investment. By doing this, we find

$$t_{11} - \frac{\left(x_{11} - \theta_1\right)^2}{2} \geq t_{12} - \frac{\left(x_{12} - \theta_1\right)^2}{2} - k_1$$

$$t_{12} - \frac{\left(x_{12} - \theta_1\right)^2}{2} \geq t_{11} - \frac{\left(x_{11} - \theta_1\right)^2}{2} - k$$

The desire for supervisor to collude with agent in production is totally destroyed through arrangement that the principal makes investment decision on the behalf of the supervisor in order to coordinate their conflicts. If the increased profit from production exceeds the loss of profit in investment, it is indeed optimal to balance the two incentives by this way. Otherwise, it is optimal for the principal to deal with the two incentives as the single task.

When the interest conflict is not serious, or the supervisor has little knowledge about production, it seems that the best choice is utilizing information in investment at the cost of production. By this means, we find that the effort of the agent in the state 21 is as the same as it in the state 22. Though the desire for collusion in production and the cost for utilizing information are both reduced, it is yet not worth utilizing information in investment as it still costs much under the conflicting interest relations. Therefore, it is the best choice for the principal to deal with the incentives independently.

In fact, when the interaction of the two incentives is substitute in the conflicting relation, the equilibrating allocation is:

$$z_{ij} = (0,1,0,1)' \qquad \pi = \pi_{\mathrm{NS}} + \Delta M'$$

$$e_{11} = e_{12} = 1 \qquad \mathrm{e}_{21} = 1 - \Delta\theta - \frac{q}{\Delta\theta}\mathrm{k}_1 \qquad \mathrm{e}_{22} = 1 - \Delta\theta + \frac{(1-q)}{\Delta\theta}k_1 \qquad \text{for } \mathrm{k}_1 < \Delta r_1$$

$$e_{11} = e_{12} = 1 \qquad \mathrm{e}_{21} = 1 - \Delta\theta - \frac{q}{\Delta\theta}\mathrm{k}_1 \qquad \mathrm{e}_{22} = 1 - \Delta\theta + \frac{(1-q)}{\Delta\theta}k_1 \qquad \text{for } \mathrm{k}_1 < \Delta r_1$$

The principal makes full use of the supervisor's information in production at the cost of investment.

Now, it is time for us to study the effect of the coordination of the two incentives on the delegation. As for this, we mean when the delegation in an organization is outcome of the coordination between the two incentives, or when the delegation can reach the goal of the coordination. Just for this reason, it is not required that decentralization strictly dominates centralization when delegation occurs, but it is optimal. The first delegation grants the supervisor partial authority in production and the second delegation full authority, we conjecture that different authority, as a result of the different delegation, is vital when they play the role of the coordination between the two incentives. The following two theorems make sure this conjecture.

Theorem2: When the two incentives interact with each other, the first delegation can reach the goal the coordination of the two incentives and the supervisor will be granted for partial authority in production; the coordination of the two incentives has no effect on the first delegation when it is independent.

The theorem 2 manifests that the first delegation can always reach the goal of coordination when the two incentives affect each other. The centre mechanism for the first delegation to coordinate incentives is that the supervisor is only granted for partial authority in production as well as the coordination between the two incentives is essentially the trade-off between the uses of supervisor's information in the two tasks. In the process of production, even if the first delegation occurs, the principal can still control the use of the supervisor's information in production decision indirectly, by controlling the target output and the total wage paid for the supervisor in production. Through this means, the principal can control the effort of the agent under the first delegation. Meanwhile in investment, how to utilize the supervisor's information depends on the principal under complete contract. So all the same, the principal can still

control the utilizing information in the tasks by the first delegation.

According to proof in appendix, we find the grand contract between the principal and the supervisor is delegation-proof if and only if:

$$t_{1j}-\frac{\left(x_{1j}-\theta_1\right)^2}{2}+\left[1-z_{1j}\right]k_j \geq t_{ij}-\frac{\left(x_{ij}-\theta_1\right)^2}{2}+\left[1-z_{ij}\right]k_j$$

$$t_{2j}-\frac{\left(x_{2j}-\theta_2\right)^2}{2}+\left[1-z_{2j}\right]k_j-\frac{p_{1j}}{p_{2j}}\frac{\Delta\theta}{2}\left[2x_{2j}-\theta_2-\theta_1\right]$$

$$\geq t_{ij}-\frac{\left(x_{ij}-\theta_2\right)^2}{2}+\left[1-z_{ij}\right]k_j-\frac{p_{1j}}{p_{2j}}\frac{\Delta\theta}{2}\left[2x_{ij}-\theta_2-\theta_1\right]$$

$$S_{ij} \geq 0, \qquad \text{i,j=1,2}$$

It is the same as the coalition incentive constrains in a centralized organization if $\lambda_j = p_{1j}$. In equilibrium, $\lambda_j = p_{1j}$ is one possible solution for those multipliers. It makes clear that the principal loses control over the agent under the first delegation, but he can control the agent directly and implement the optimal collusion-proof grand mechanism.

The conclusion of the theorem 1 is similar to the Equivalence Principle (Grimaud, Laffont and Martimort, 2003) because the centre mechanism for them is that the principal certainly is the final master of the information utilizing and the agent under the first delegation. However, the first delegation under multitask is the outcome of the coordination while it is collusion. Moreover, it is possible that the first delegation can not reach the aim of the coordination if $k \neq (\Delta\theta)^2$. The reason is that the coordination between the two incentives is also influenced by the relative importance of the two tasks.

Theorem3: When $\alpha > 1$, $\beta = -1$, if $2\Delta M > k$ and $k_1 > \Delta r_1$, then the second delegation reach the goal of the coordination of the two incentives, and supervisor will be granted for full authority in investment; the coordination of the two incentives has on effect on the second delegation at any other condition.

Theorem 3 shows that coordinating effect on the second delegation, and the second delegation can only reach the goal to coordinate the two incentives when the

interest relations are conflicting. The central mechanism lies that the supervisor is granted for full authority in investment by the second delegation, as a result of it, the principal at least loses some benefit in investment. So the second delegation can only achieve the goal of the coordination when the interaction of the two incentives is substitute. However, it must be required that the second delegation can destroy the collusion between the supervisor and the agent, and the benefit from it must exceed the cost if the second delegation is indeed optimal.

In fact, we can make clear the mechanism for the second delegation to coordinate the two incentives by discussing its conditions more in detail. If the second delegation occurs when $\alpha < 1,\ \beta = -1$ and $k_1 > \Delta r_1$, then

$k > k_1 > \Delta r_1 > 2\Delta M$,

k is the minimal cost for the second delegation; because ΔM is the increased profit from the supervisor's information in production without collusion, it is maximal benefit from the second delegation; as $\alpha < 1$, the cost for the second delegation must exceed the benefit. Since α is not only means the interests conflicts between the supervisor and the principal, but also the relative value between them, then we can conclude that the second delegation must not be optimal, or coordinate the two incentives if the benefit for supervisor from the delegation is less than the cost for principal, given the specific interest relations.

If the second delegation occurs when $\alpha > 1,\ \beta = 1$ and $2\Delta M' > k$, but then the coalition incentive constrains in state θ_1 is

$$t_{11} - \frac{e_{11}^2}{2} = t_{12} - \frac{e_{12}^2}{2}$$

Under the interest relations, the supervisor always takes the investment action a_2 under the second delegation, the desire for the supervisor to collude with the agent has not yet changed under this delegation, and the benefit from it is zero.

When $\alpha > 1, \beta = -1$ and $2\Delta M' > k$, if the second delegation occurs as $k_1 < \Delta r_1$, then the probability for the supervisor and the agent to collude is

$$\frac{2q-1}{q} - \frac{k_1(1-q)}{\Delta\theta^2}$$

Δr_1 is the difference of the information rent for the agent in state 12 and 11, and it also means that the maximal benefit for the supervisor to collude with the agent. The second delegation results in the collusion between the supervisor and the agent as $k_1 < \Delta r_1$. Moreover, the principal obtains the benefit of k at the expense of k_1 under the second delegation while only k under a centralized organization. The central reason for it is the second delegation grants the supervisor full authority in investment and the result of it is that the principal loses some control in production and full control in production because the supervisor's information in production is correlated with it in investment.

Compared with the first delegation, the additional condition for the second delegation is $k_1 > \Delta r_1$, and it ensures that the collusion between the supervisor and the agent is impossible under the second delegation as the supervisor obtains full authority in investment.

Based on those theorems, we study the comparative effect of the interest conflicts α and the supervisor's knowledge q on the coordination and delegation. Because the interest conflicts and the supervisor's knowledge indirectly affect how the delegation achieves the goal of the coordination between the two incentives by directly influencing the coordination, we focus on their effect on the delegation.[16]

Corollary1: When $\alpha < 1, \beta = 1$, the interest conflicts have no effect on delegation; when $\alpha > 1, \beta = -1$, the first and second delegation are more likely to coordinate the two incentives in optimal way with the interest conflicts increasing.

When $\alpha < 1, \beta = 1$, the interest conflicts do not affect the nature of the coordination between the two incentives though it does affect the information utilizing and the profit under multitask. Then the first delegation can achieve the goal of the coordination while the second delegation not as these two delegations grant different authority for the supervisor, so the interest conflicts has no effect delegation under the interest relations.

When $\alpha > 1, \beta = -1$, the cost for the principal to induce the supervisor to tell truth in investment is increasing, or the direct communication between the principal and the

[16]As for comparative static, we only deal with the comparative effect of the interest conflicts and the supervisor's knowledge on whether the delegation can attain the goal of the coordination in equilibrium, that is, whether or not the variation of them influences that the delegation coordinates the two incentives in the best way.

supervisor becomes more difficulty. However, if the supervisor has enough knowledge about the production, the principal can benefit from it by utilizing the supervisor's information in production and increasing its profit at the cost of investment. When the interest conflicts become more serious, the interaction of the two incentives is more likely to be substitute, so the first and the second delegation is more capable of coordinating the two incentives in the best way.

Corollary2: When $\alpha < 1, \beta = 1$, the supervisor's knowledge about production has no effect on delegation; when $\alpha > 1, \beta = -1$, the first delegation is more likely to best means to coordinate the two incentives with the increase of the interest conflicts, but its effect the second delegation is ambiguous.

The same to the corollary 1, the supervisor's knowledge does not affect the nature of the coordination between the two incentives when $\alpha < 1, \beta = 1$, and it has no effect on delegation.

When $\alpha > 1, \beta = -1$, the interaction of the two incentives is more likely to be substitute because the principal can make more profit by utilizing the supervisor's information at the expense of investment. Since it can always accomplish the goal of the coordination when the two incentives affect each other, the first delegation can coordinate the two incentives better with the supervisor's knowledge increasing.

However, the same logic can not go through on the second delegation for it grants the supervisor full authority in investment. As the principal can not control the information utilizing in production as well as loses the control in investment, it is necessary for the second delegation to coordinate the two incentives optimally that the supervisor has no incentive to collude with the agent, that is, $k_1 > \Delta r_1$. Hereby, when $k_1 > \Delta r_1$ is still satisfied with the increase of the supervisor's knowledge, the second delegation is more likely to be best way to coordinate the two incentives, otherwise, it may be less to the best way if $k_1 < \Delta r_1$.

An interesting question is whether the principal always prefers the coinciding interest relation to the conflicting one if he can freely choose the interest relations. In more accurate word, it means that given the knowledge of the supervisor, what the kind of the interest relations is best for the principal when the interest conflicts between

principal and supervisor is variable.

Corollary3: If $k < \min\{\Delta r, \Delta M\}$, then principal prefers the conflicting interest relation to the coinciding one, otherwise the coinciding one.[17]

$k < \Delta r$ means that the principal can not make full use of the supervisor's information in production under the coinciding interest relations. $k < \Delta M$ is that the profit under the coinciding interest relations is less than it in the conflicting interest relations if the principal can make full use of the supervisor's information in production. Nevertheless, the principal can fully utilize the supervisor's information in production by making the profit in production at the cost of investment under the conflicting interest relation when the interest conflicts are variable. Therefore, if $k < \Delta r$ and $k < \Delta M$, the conflicting interest relations are the most preferred for principal because the principal can make more effective use of the supervisor's information in production under the conflicting interest relations than it under the conflicting one. Otherwise it is the coinciding one. By the similar argument, we can discuss given the interest conflicts, what is the preferred interest relations for the principal when the supervisor's knowledge about the production is variable, the mechanism and the conclusion are similar to Grimaud, Laffont and Martimort (2003).

4. Collusion, Coordination and Delegation under Incomplete Contract

4.1 The General Collusion-proof Principle

Following the tradition of cheap talking, we also discuss under incomplete contract, which means that principal is unable to make credible commitment in choosing specific investment action[18]. Given the contract is incomplete; the collusion-proof principle can not work now. However, the logic of it is still in effect that is principal can get the supervisor's information and utilize it by the sequential

[17] Even if we consider the middling interest relation, this conclusion will not change. But if the two tasks are not the same important, that is $k \neq (\Delta\theta)^2$, then the preferred interest relation has not only to do with the supervisor's knowledge, but also the relative importance of the two tasks.

[18] If the transfer between principal and supervisor is also forbidden, then the conclusions about coordination are not changed as compared to commitment problem, but conclusions about delegation change because the first delegation is not possible.

arrangement of different tasks. At the same time, the supervisor and the agent consider not only the influence it will exert on the present task, but also that of the future ones when reporting the information to the principal, since the principal would make use of the information got from the present task in the following one. Therefore, we have to employ the sequential equilibrium to analyze the issue of collusion. Similar to complete contract, we should only focus on the direct mechanism under incomplete contract, while the point differs from the complete contract is that here it may be unworkable to induce the two to tell the truth, what's more, their collusion relates to the time sequence of different tasks.

4.1. A. The General Revealing Mechanism in Production

If principal carries out production decision first, then principal and supervisor both consider its effect on the succeeding investment decision. According to the conclusion made by Craworf and Sobel (1982), there is no essential information exchange in the cheap communication between the principal and the supervisor.[19] As a result, what principal will choose is absolutely determined by what know in production decision. As for the same reason, the supervisor and the agent would take its influence in investment into consideration when they report the information to the principal in production. Due to conclusion of Grimaud, Laffont and Martimort (2003), we know that centralization weakly dominates decentralization at this time. Thus, the collusion mechanism between supervisor and agent we need consider is as follows:

$$\underset{\{\phi,y\}}{Max}\sum P_{ij}[t(\phi_{ij})-y_{ij}+[1-z(u\circ\phi_{ij})]k_j]$$

$$s.t. \quad y_{1j}-\frac{[x(\phi_{1j})-\theta_1]^2}{2}\geq y_{2j}-\frac{[x(\phi_{2j})-\theta]^2}{2}$$

$$y_{1j}-\frac{[x(\phi_{1j})-\theta_1]^2}{2}\geq \bar{U}_{1j} \quad \text{j=1,2}$$

The side contract of the supervisor and agent is similar to that in complete contract, which maximize supervisor profit with agent participating constrains and incentive-compatible constrains. The difference is that investment decision is

[19] If we consider the transfer between principal and supervisor, principal will get supervisor's information about investment under harmonizing interest relation, but it does not affect our conclusion in this section.

accomplished by the information that the principal gets from the production decision. Herein, $u \circ \phi_{ij}$ represents the posterior belief of principal towards the state when the state of the supervisor reporting to him is ϕ_{ij}. $z(u \circ \phi_{ij})$ represents that given that posterior belief, the probability of the action a_1 that will be taken by the principal in investment . And as for the rest of the symbols, what they represent are the same as they in collusion-proof mechanism under complete contract.

4.2. B. The General Revealing Mechanism in Investment

If principal implements investment decision first, principal and supervisor both consider its effect on sequential production too. In investment, if the second delegation does not emerge, the cheap talking between principal and supervisor that is analogue to cheap talking in single task, does not transmit any information from supervisor to principal because the more information supervisor sends to principal in investment, the less information rent that supervisor and agent will get in production under multiple tasks. So in this time, what we should pay attention to is the collusion mechanism between supervisor and agent when second delegation occurs in investment, that is:

$$\underset{\{a,y\}}{Max} \sum P_{ij}[W(u(a_k),\ \theta_i) - y_{ij} + k_j(a_k)]$$

$$s.t. \quad y_{1j} - \frac{[x(u(a_k),\theta_1) - \theta_1]^2}{2} \geq y_{2j} - \frac{[x(u(a_k),\theta_1) - \theta_1]^2}{2}$$

$$y_{1j} - \frac{[x(u(a_k),\theta_1) - \theta_1]^2}{2} \geq \bar{U}_{1j}$$

$$y_{2j} - \frac{[x(u(a_k),\theta_2) - \theta_2]^2}{2} \geq \bar{U}_{2j} \quad \text{j=1,2}$$

Here $u(a_k)$ represents that if the supervisor chooses to take action a_k, then the principal forms the posterior belief about state ij; $W_{ij}(u(a_k)\ \theta_i), x(u(a_k)\ \theta_i)$ represents that in this posterior belief, the wage and output that principal sets for agent when report his type in the following production.; and $k_j(a_k)$ is the benefit from investment when supervisor chooses to take action a_k at the state of ij. And the rest of the symbols have the same meanings as they in collusion-proof principal in complete contract. Here the means taken by the supervisor and the agent in their collusion is not to control the signals sent to the principal, but to control the investment action

supervisor takes.[20]

If the posteriori belief of the principal is $u(a_k)$, wages and output for agent in production must satisfy:

$$W_{1j}(u(a_k)\ \theta_1) - -\frac{\left[x_{1j}(u(a_k)\ \theta_1)-\theta_1\right]^2}{2} \geq W_{2j}(u(a_k)\ \theta_2) - \frac{\left[x_{2j}(u(a_k)\ \theta_2)-\theta_1\right]^2}{2}$$

$$W_{2j}(u(a_k)\ \theta_2) - \frac{\left[x_{2j}(u(a_k)\ \theta_2)-\theta_2\right]^2}{2} \geq 0$$

These equalities are binding in equilibrium, and agent gains the information rent in the state of θ_1 while the reserving utility in the state of θ_2. Accordingly, in the state of θ_2 the agent is unable to bribe the supervisor and the supervisor will always choose the investment action in benefit of himself, while in the state of θ_1 collusion may occur because the agent receives the information rent and the amount of the information rent is related to the supervisor's action a_k.

4.2. Coordination and Delegation

Similar to the analysis in complete contract, now we start to discuss coordination and delegation in an organization.

Theorem4: When $\alpha < 1, \beta = 1$, the interaction of the two incentives is complementary; when $\alpha > 1, \beta = -1$, if $2\Delta M'' > k$, the interaction of incentives is substitute, otherwise the interaction of the two incentives is independent. $\Delta M''$ is:

$$\Delta M'' = \begin{cases} \dfrac{q(1-q)}{4\Delta\theta^2}k_1^2 - \dfrac{q(1-q)}{2\Delta\theta^2}kk_1 + \dfrac{2q-1}{2}k & k_1 \leq \Delta r_1 \\ \Delta M & k_1 > \Delta r_1 \end{cases}$$

Similar to the theorem 1, the interaction of the two incentives is essential the trade-off between utilizing the supervisor's information in both tasks, and it is mainly determined by the interest relation between principal and supervisor as well as is affected by the interest conflict between principal and supervisor and supervisor's knowledge about production. However, what differs from theorem 1 is that the principal

[20] Unlike implementing production first, maybe decentralization does not dominate centralization in investment at this time. Nevertheless, when decentralization does not dominate centralization in investment, the interaction of the two incentives is independent, so by only consider collusion mechanism under second delegation does not change our conclusion at this time. Similarly, we does not need take into account the information that principal will get from supervisor in sequential production.

no longer make credible commitment and sequential arrangement of tasks becomes crucial for coordination between the two incentives. When the interest relation between principal and supervisor is harmonizing in an organization, principal can reduce the desire for supervisor to collude with agent in production through putting production in effect first, and then use supervisor's information in both tasks. The reason lies in that when principal carries out the succeeding investment decision in interest of him, it also benefit supervisor because they have more common benefit than conflicting even if principal is lack commitment in investment. Thus by arranging production decision first, principal can still make the same effective use of the supervisor's information under incomplete contract as complete contract.

However, the same logic can not go through under conflicting interest relation. If production decision is conducted first, the desire for supervisor to collude in production is enlarged than single task because the truth-telling in production will also result in loss in succeeding investment decision. The reason is that when he implements investment decision, principal always acts for the sake of himself if he lacks commitment in investment, which brings about loss for supervisor in investment. If the investment decision is made first, the only way for principal to coordinate the two incentives is the second delegation from analysis about general revealing mechanism. At the same time, the second delegation is at least at cost of principal's benefit in investment, so it can occur in equilibrium only if that the second delegation will result in profit in production is greatly increased, that is:

$$2\Delta M'' > k$$

So when he lack commitment in investment, principal can only coordinate the two incentives by arranging investment and the second delegation under conflicting interest relation.

After analyzing the interaction of the two incentives in the organization, we will then focus on how coordination in the multitask directly determine the delegation in the organization

Theorem5: when $\alpha < 1, \beta = 1$, the first delegation will occur and the supervisor will be granted for part authority in production; otherwise the coordination of the two

incentives have not any effect on the first delegation under incomplete contract.

The central of theorem 5 are similar to those of theorem 2: supervisor is only granted for part authority in production and principal can still control the use of information in production when the first delegation. However, because principal lacks commitment in investment, the first delegation can reach the goal of coordination of the two incentives when it does not need commitment in investment while it can not arrive at the goal when it needs. Therefore, the coordination can still result in the first delegation under harmonizing interest relation while not under conflicting interest relation.

Theorem6: when $\alpha > 1, \beta = -1$ and $2\Delta M'' > k$, the second delegation will occur and supervisor will be granted for full authority in investment; otherwise the coordination do not have effect on the second delegation.

The same to theorem 3, due to the second delegation resulting in full authority in investment, it could only occur in conflicting interest relations. Nevertheless, since the principal can not make credible commitment, the second delegation is the only commitment arrangement for principal not utilizing information in investment. So it is not needed for supervisor not collude with agent in production, that is, $k_1 > \Delta r_1$ is necessary for the second delegation. In fact, we find when $k_1 < \Delta r_1$, if it is satisfied with:

$$\frac{q(1-q)}{4\Delta\theta^2} k_1^2 - \frac{q(1-q)}{2\Delta\theta^2} k k_1 + \frac{2q-1}{2} k > k/2$$

The second delegation in investment dominates centralization, at the same time, supervisor collude with agent and gets information rent in production from agent.

Similar to theorem 3, when $\alpha < 1, \beta = -1$, the second delegation will not occur in equilibrium because the benefit for supervisor from it is less than the cost for principal; when $\alpha > 1, \beta = 1$ and $2\Delta M'' > k$, neither will second delegation for supervisor always prefers the same action in investment, and the second delegation can not reduce the desire for supervisor to collude with agent in production.

On the basis of the theorem 5 and theorem 6, we also analyze the comparative conclusions under incomplete contract.

Corollary4: the change of the interest conflict between principal and supervisor exerts no influence on the first delegation whatever interest relation is; when $\alpha > 1, \beta = -1$, the increase of the interest conflict increases the occurring of the second delegation.

The same to complete contract, the interest conflict has not any effect on coordination and the first delegation under harmonizing interest relation by conducting production decision first. Meanwhile, the first delegation can not reach the goal of coordination of the two incentives under conflicting interest relation for it needs principal's commitment in investment, so variation of interest conflict does not affect its occurring.

When $\alpha > 1, \beta = -1$, the only means for principal to coordinate the two incentives is the second delegation due to his lacking commitment. Then, with increasing in interest conflict between principal and supervisor, the cost for supervisor to collude with agent increases, which reduces the desire for supervisor to collude, and principal obtains more information about production by second delegation. In fact, as the second delegation occurs, the possibility of supervisor colluding with agent is:

$$x = \max\left\{\frac{2q-1}{q} - \frac{k_1(1-q)}{\Delta\theta^2}, 0\right\}$$

Hence, the increase in interest relations resulting in principal getting more information about production from the second delegation by reducing the possibility of collusion will help the occurring of the second delegation.

Corollary5: when $\alpha < 1, \beta = 1$, the change of the supervisor's knowledge has no influence on the first delegation; when $\alpha > 1, \beta = -1$, the effect of the increase of the supervisor's knowledge on the second delegation is ambiguous.

When $\alpha < 1, \beta = 1$, the principal still can coordinate the two incentives the same well as complete contract by executing production decision first even if he lacks commitment in investment.

When $\alpha > 1, \beta = -1$, there are two effects with increasing in supervisor's knowledge about production. One is productive effect, which is that principal can get more information from supervisor's investment choice and increase the profit in

production with the increase of supervisor's knowledge; the other is collusive effect, which is that the increase in supervisor's knowledge cause difference of information rent for agent in different state increases, and desire for supervisor to collude with agent increases, which increases the possibility to collude. Therefore, the effect on the second delegation is determines which effect is dominant. If the condition satisfies $k_1 > \Delta r_1$, then increase in supervisor's knowledge helps to the second delegation because there is only productive effect. And when $k_1 < \Delta r_1$, the effect for increase in supervisor's knowledge is

$$\frac{d\Delta M''}{dq} = \frac{k_1^2(1-2q)}{4\Delta\theta^2} + \frac{kk_1(2q-1)}{2\Delta\theta^2} + k$$

So when the interest conflict is serious, increase in supervisor's knowledge may hinder the occurring of the second delegation.

5. Extension and discussion

Our main conclusion is still proper when we consider some extensions. If the limited liability of supervisor is variable, that is:

$$S \geq A, \quad 0 \geq A > -\Delta r$$

Here A stands for the limited liability. All the conclusions in text can be reached except that the necessary and sufficient condition for the interaction of the two incentives is:

$$\pi_{NS} + \Delta(-A) + k/2 \qquad \Delta(-A) = \frac{-A(1-q)\left[(4q-2)\Delta\theta^2 + (1-q)^2\Delta\theta^2 A\right]}{4q\Delta\theta^2}$$

If the information structure changes and the agent only know the information of his own type, would the conclusions change? Now collusion mechanism between supervisor and agent has transformed into the informed principal problem. According to Maskin and Tirole(1990,1992), the collusion mechanism is the same to it in this paper, so the conclusions in the paper have not changed at all. Another question is about whether the bargaining power of supervisor in collusion will affect the conclusions in this paper. From the conclusion of Grimaud, Laffont and Martimotr(2003), we know

that centralization weakly dominates all kinds of decentralization. So the main factors for coordination between the two incentives yet not change at all, so we feel confident to conjecture that main conclusions in this paper are holding. We can also consider the relative importance of the two tasks with change of k, the main does not change either. However, we will find that the second delegation will more likely to occur with the decrease of importance of the task of investment.

Maybe the most interesting extension is to discuss coordination and delegation with more complicated interest relation. If we consider three states and three choices in the task of investment, the interests are three-dimensional. In three-dimensional interest relations, there are not only common interests we have to analyze but also the amount of common interest, i.e. in which state the common interests exist. The discussion on conflicts of interests does not only involve the level of conflict, but also the problem in which state there are conflicts of interests. Compared with the bi-dimensional relations in this paper, the three-dimensional interests make the collusion-proof more difficult because it is necessary to avoid more collusion-temptation. Nevertheless, the basic trade-off is still the utilizing the information between the two tasks, and decisive forces for coordination are interest relation between principal and supervisor, interest conflict and the supervisor's knowledge about the task of production, so we think our ideas will work under this more general setting.

6. Conclusion

In this paper, we deal with the coordination and delegation under multitask with cheap talking and collusion. As the information made use of in one task is correlated with that in the other, so there exists the trade-off between utilizing the supervisor's information in different tasks. The interest relations between principal and supervisor determine the desire for supervisor to collude in production as principal use supervisor's information in the task of production as well as the cost use supervisor's information in the task of investment, thus it is the main force to decide the interaction of the two incentives; the interest conflict decides the cost for principal to utilizing

supervisor’s information in investment; the supervisor’s knowledge about production decides the value to utilizing the supervisor’s information in production. These three factors altogether determine the interaction of the two incentives under complete contract. However, the sequential arrangement of the two tasks is also necessary to coordinate the two incentives when principal is short of commitment in investment in order to control the desire for supervisor to collude besides these three factors. As the interest relation is harmonizing, the desire for supervisor to collude in the task of production and cost for principal use supervisor’ information in the task of investment is less than single task, so principal utilizing the supervisor’s information in both tasks and the coordination of the two incentives is complementary. The desire for supervisor to collude is greatly increased compared with single task when principal utilize supervisor’s information in investment under conflicting interest relation, so the possible efficient way for coordination utilizes supervisor’s information only in the task of production while ignores the task of investment in order to decrease the desire for supervisor to collude, then the coordination of the two incentives may be substitute under conflicting interest relation.

As for delegation, the first delegation grants part authority for supervisor in the task of production and the second delegation grants full authority for supervisor in investment. Since principal still control the use of supervisor information in production and the desire for supervisor to collude by the first delegation, it can always reach the goal of coordination under complete contract, so the coordination will always result in the first delegation under complete contract if the two incentives interact with each other. However, the first delegation can not always control the desire for supervisor to collude and reach the aim of coordination when principal lacks commitment in investment. It can arrive at the goal of coordination under harmonizing interest relation because commitment is not necessary for coordination, so the coordination under harmonizing interest relation still bring about the first delegation under incomplete contract. At the same time, the coordination between incentives has not any effect on the first delegation for commitment in investment is necessary for coordination. Since the first delegation always reach the goal of coordination under harmonizing interest

relation while it can reach the goal only when interest conflict and supervisor's knowledge are high enough. Therefore, we find that harmonizing interest relation is more likely to result in the first delegation than conflicting one.

As for the second delegation, when it occurs, principal does at least not utilize supervisor's information in investment, which can only emerge in equilibrium when the coordination between the two incentives is substitute. So it could only occur under conflicting interest relation, but for it emerges in equilibrium, the second delegation must result in that the increase of principal's profit is high enough to make up for his loss in investment from second delegation. Here, we find complete difference between the authorization of multiple tasks and solo task as for the increase of the interest conflict. This is because the authorization of multiple tasks has to take the coordination of other incentives into account, and it is the key factor for delegation while the increase of the interest conflict may make the coordination of the two incentives more easily in an organization.

Appendix

Appendix 1

In this appendix, we only deal with as $\alpha < 1, \beta = 1$ and $\alpha > 1, \beta = -1$, but lemma 1 is still applied even if we consider other interest relations. We will use lemma1-6 to prove all the conclusions of the thesis, with lemma1-4 to complete contract and lemma 5-6 to incomplete contract.

Lemma1: Given any interest relations, centralization always dominates or weakly dominates any kinds of decentralization in an organization.

Proof: According to the conclusions of Grimaud, Laffont and Martimort(2003), here the collusion-proof-principle works, and the grand mechanism is collusion-proof if and only if[21] :

$$t_{1j} - \frac{\left(x_{1j} - \theta_1\right)^2}{2} + \left[1 - z_{1j}\right]k_j \geq t_{ij} - \frac{\left(x_{ij} - \theta_1\right)^2}{2} + \left[1 - z_{ij}\right]k_j$$

[21] The proof of collusion-proof-principle is the same to Grimaud, Laffont and Martimort expect for that we assume supervisor limited liability, so it still work here and the reader can see detail in their paper.

$$t_{2j} - \frac{\left(x_{2j} - \theta_2\right)^2}{2} + \left[1 - z_{2j}\right] k_j - \frac{\lambda_j}{P_{2j}} \frac{\Delta\theta}{2} \left[2x_{2j} - \theta_2 - \theta_1\right]$$

$$\geq t_{ij} - \frac{\left(x_{ij} - \theta_2\right)^2}{2} + \left[1 - z_{1j}\right] k_j - \frac{\lambda_j}{P_{2j}} \frac{\Delta\theta}{2} \left[2x_{ij} - \theta_2 - \theta_1\right]$$

$$P_{1j} = \lambda_j + u_j \qquad \lambda_j, u_j \geq 0, \qquad i,j=1,2$$

λ_j represents incentive-compatible Lagrange multiplier stimulated by the type θ_1, μ_j represents the participating Lagrange multiplier of the type θ_1.

If these two kinds delegations occur at the same time, the profit the principal gets is at most $k/2$ in investment , while in production the contract between supervisor and agent is delegation-proof if and only if:

$$t_{1j} - \frac{(x_{1j} - \theta_1)^2}{2} \geq t_{ij} - \frac{(x_{2j} - \theta_1)^2}{2}$$

$$t_{2j} - \frac{(x_{2j} - \theta_2)^2}{2} - \frac{p_{1j}}{p_{2j}} \frac{\Delta\theta}{2} \left(2x_{2j} - \theta_1 - \theta_2\right) \geq$$

$$t_{ij} - \frac{(x_{ij} - \theta_2)^2}{2} - \frac{p_{1j}}{p_{2j}} \frac{\Delta\theta}{2} \left(2x_{ij} - \theta_1 - \theta_2\right) \qquad i,j=1,2$$

If only the first kind delegation occurs and supervisor is granted for partial authority in production, the side contract between the supervisor and the agent is delegation-proof if and only:

$$t_{1j} - \frac{(x_{1j} - \theta_1)^2}{2} + (1 - z_{1j}) k_j \geq t_{ij} - \frac{(x_{ij} - \theta_1)^2}{2} + (1 - z_{ij}) k_j$$

$$t_{2j} - \frac{(x_{2j} - \theta_2)^2}{2} + (1 - z_{2j}) k_j - \frac{p_{1j}}{p_{2j}} \frac{\Delta\theta}{2} (2x_{2j} - \theta_1 - \theta_2) \geq$$

$$t_{2j} - \frac{(x_{ij} - \theta_2)^2}{2} + (1 - z_{ij}) k_j - \frac{p_{1j}}{p_{2j}} \frac{\Delta\theta}{2} (2x_{ij} - \theta_1 - \theta_2)$$

Based on these, we can easily prove that centralization weakly dominates all other kinds' decentralization in an organization.

Lemma2: When $\alpha < 1, \beta = 1$, in equilibrium z_{ij} is $(1,0,1,0)'$ and the profit that

principal get is $\pi_{NS} + \Delta(k_1 - A) + k - k_1/2$; when $\alpha > 1, \beta = -1$, if $2\Delta M' - 2\Delta(-A) > k$, z_{ij} is $(0,1,0,1)'$ and the profit is $\pi_{NS} + \Delta M'$ in equilibrium.

Proof: we apply the same approach of Grimaud, Laffont and Martimort (2003), first considering the binding constrains and then proving the other constrains no binding in equilibrium.

First we the optimization problem for principal that are constrained by:

$$t_{11} - \frac{(x_{11} - \theta_1)^2}{2} + [1 - z_{11}]k_1 \geq t_{12} - \frac{(x_{12} - \theta_1)^2}{2} + [1 - z_{12}]k_1$$

$$t_{12} - \frac{(x_{12} - \theta_1)^2}{2} + [1 - z_{12}]k_2 \geq t_{22} - \frac{(x_{22} - \theta_1)^2}{2} + [1 - z_{22}]k_2$$

$$t_{21} - \frac{(x_{21} - \theta_2)^2}{2} + [1 - z_{21}]k_1 - \frac{\lambda_1}{P_{21}}\frac{\Delta\theta}{2}[2x_{21} - \theta_2 - \theta_1]$$

$$\geq t_{22} - \frac{(x_{22} - \theta_2)^2}{2} + [1 - z_{22}]k_1 - \frac{\lambda_1}{P_{21}}\frac{\Delta\theta}{2}[2x_{22} - \theta_2 - \theta_1]$$

$$t_{21} - \frac{(x_{21} - \theta_2)^2}{2} \geq 0 \qquad t_{22} - \frac{(x_{22} - \theta_2)^2}{2} \geq 0$$

$$0 \leq \lambda_1 \leq p_{11}$$ [22]

Following the constrain 3, we know that $\lambda_1 = p_{11}$ in equilibrium. Apparently as $\alpha < 1, \beta = 1$, it must be required that $z_{11} = 1$. If not, we can construct another allocation based on the original allocation by means of:

$z_{11}' = 1 \quad t_{11}' = t_{11} + (1 - z_{11})k_1$ and for other allocation

$z'_{ij} = z_{ij}$

By this principal can get more profit than before, so it must be satisfied that $z_{11} = 1$.

In the same way, we find that $z_{21} = 1$ in equilibrium under $\alpha < 1, \beta = 1$。Adding these conditions, we can solve the optimization problem constrained by the above five inequalities. The profit for principal is

[22] In order for simplification, we do not consider the supervisor's limit liability constrain is variable, but the proof can easily extended to such circumstance.

$$\pi_{NS} + \Delta\left[(1-z_{22})k_1\right] + k/2 + p_{12}(1-z_{12})k + p_{22}(1-z_{22})k$$
$$-p_{11}\left[1+(z_{12}-z_{22})\right]k_1 - p_{12}(z_{12}-z_{22})k - p_{21}(1-z_{22})k_1$$

$$\Delta\left[(1-z_{22})k_1\right] = \begin{cases} \dfrac{k(1-z_{22})(1-q)[(4q-2)\Delta\theta^2 - (1-q)^2 k_1(1-z_{22})]}{4q\Delta\theta^2} & \mathrm{k}_1(1-z_{22}) \le \Delta r \\ \Delta M & \mathrm{k}_1(1-z_{22}) > \Delta r \end{cases}$$

So it can be concluded that $z_{12} = z_{22} = 0$ in equilibrium and the profit is

$\pi_{NS} + \Delta(k_1) + k - k_1/2$

By setting $\lambda_2 = p_{21}$ together with the five above five binding constrains, we can prove the all other collusion-proof constrains are satisfied with, therefore the solution above optimization problem is equilibrium.

Similarly, when $\alpha > 1, \beta = -1$, if $2\Delta M' - 2\Delta(-A) > k$, then $z_{ij} = (0,1,0,1)'$ and the profit is $\pi_{NS} + \Delta M'$.

Lemma3: when the two processes or incentives is on independence, the first kind delegation will occur and supervisor will be granted for part authority in production; otherwise the coordination between the two incentives has no effect on the first kind delegation.

Proof: In order to analyze whether the first kind delegation occurs in equilibrium, the only thing that we need to do is confirm whether the allocation in equilibrium can be reproduced under the first kind delegation. Equivalently, that is whether the allocation in equilibrium is still collusion-proof when $\lambda_i = p_{i1}$, due to the conclusion of Grimaud, Laffont and Martimort(2003).

Following the proof of lemma 2, we know that as $\alpha < 1$, β=1, these constrains are binding in equilibrium:

$$t_{11} - \frac{e_{11}^2}{2} \ge t_{12} - \frac{e_{12}^2}{2} + k_1$$

$$t_{12} - \frac{e_{12}^2}{2} \ge t_{22} - \frac{(e_{22} - \Delta\theta)^2}{2}$$

$$t_{21}-\frac{e_{21}^2}{2}-\frac{q}{1-q}\frac{\Delta\theta}{2}(2e_{21}-\Delta\theta)\geq t_{22}-\frac{e_{22}^2}{2}-\frac{q}{1-q}\frac{\Delta\theta}{2}(2e_{22}-\Delta\theta)+k_1$$

$$t_{21}-\frac{e_{21}^2}{2}\geq 0 \qquad \lambda_1=p_{11}$$

So, we can get as $\lambda_2=p_{21}$:

$$t_{22}-\frac{e_{22}^2}{2}+k-\frac{1-q}{q}\frac{\Delta\theta}{2}(2e_{22}-\Delta\theta)>t_{21}-\frac{e_{21}^2}{2}-\frac{1-q}{q}\frac{\Delta\theta}{2}(2e_{21}-\Delta\theta)$$

$$t_{22}-\frac{e_{22}^2}{2}-\frac{1-q}{q}\frac{\Delta\theta}{2}(2e_{22}-\Delta\theta)>t_{12}-\frac{(e_{12}+\Delta\theta)^2}{2}-\frac{1-q}{q}\frac{\Delta\theta}{2}(2e_{12}+\Delta\theta)$$

$$t_{22}-\frac{e_{22}^2}{2}+k-\frac{1-q}{q}\frac{\Delta\theta}{2}(2e_{22}-\Delta\theta)>t_{11}-\frac{(e_{11}+\Delta\theta)^2}{2}-\frac{1-q}{q}\frac{\Delta\theta}{2}(2e_{11}+\Delta\theta)$$

That is meaning that collusion-proof condition is still satisfied with as $\lambda_i=p_{i1}$, therefore, the first kind delegation also is optimal. By similar means, we can prove when the first delegation is optimal under other interest relations.

Lemma4: when $\alpha>1,\beta=-1$, if $2\Delta M-2\Delta(-A)>k$ and $k_1>\Delta r_1$, the second kind delegation will occur and supervisor will be granted for full authority in investment; under any other conditions, the coordination will not result in the second delegation.

Similar to lemma 3, the only thing we need to do is confirm whether the allocation in equilibrium can be reproduced under second kind delegation.

From lemma 2, it is known that if $2\Delta M-2\Delta(-A)>k/2$ and $k_1>\Delta r_1$, the two incentives interact with each other and z_{ij} are $(0,1,0,1)'$ in equilibrium as $\alpha>1,\beta=-1$. At this time, when the second delegation occurs, the supervisor will not collude with agent because the benefit from not collusion is always exceeding the benefit from collusion as $k_1>\Delta r_1$ is satisfied with. So under this condition, the second delegation is optimal and supervisor will be granted full authority in investment

When $\alpha>1,\beta=-1$ and $k_1<\Delta r_1$, by the second delegation, the profit that principal get is:

$$\frac{q(1-q)}{4\Delta\theta^2}k_1^2-\frac{q(1-q)}{2\Delta\theta^2}kk_1+\frac{2q-1}{2}k<\Delta M'$$

So the second delegation can not occur in equilibrium. By the similar means, we find that the second delegation is not optimal under any other interest relations.

Lemma5: When $\alpha < 1, \beta = 1$, the first kind delegation will occur and supervisor will be granted for part authority in production; under other circumstances, the coordination could not bring about the first kind delegation.

Lemma6: when $\alpha > 1, \beta = -1$ and $2\Delta M' - 2\Delta(-A) > k$, the second delegation will occur and supervisor will be granted for full authority in investment; under any other condition, the coordination will not result in second delegation.

The proof of lemma 5 and lemma is similar to lemma 3 and lemma 4, so we omit them.

Appendix 2

In this section, we simply discuss the delegation under middling interest relation, that is $\alpha < 1, \beta = -1$ or $\alpha > 1, \beta = 1$.For simplification, we only analyze the coordination between the two tasks when supervisor's limit liability is constant: $A = 0$. Theorem 2* deals with complete contract and theorem 3* with incomplete contract. .

Theorem2*: when $\alpha > 1, \beta = 1$ and if $\Delta(k_1) > \left[(1-q)(k_1 - k)\right]/2$, the interaction of the two incentives is partly substitute; when $\alpha < 1, \beta = -1$ and if $2q - 1 > \alpha$,the interaction of the two incentives is also partly substitute; in any other condition, the two incentives is independent..

Theorem3*: when $\alpha < 1, \beta = -1$, if $2q - 1 > \alpha$, the interaction of the two incentives is partly substitute, and in other circumstance, the two incentives is independent.

The proof and mechanism is the same to appendix 1, so we do not go further again.

References

1. Aghion P. and J. Tirole (1997): “Formal and Real Authority in Organizations”, *Journal of Political Economy*, 105, 1-29.
2. Arrow, Kenneth J. (1974): “The Limits of Organization”, W. W. Norton & Company, New York • London.
3. Austen-Smith, D., (1994): “Strategic Transmission of Costly Information,” *Econometrica*, 62, 955-963.
4. Baliga, S. and T. Sjostrom (1998): “Decentralization and Collusion”, *Journal of Economic Theory,* 83, 196-232.
5. Baliga，S.1999，“Collusion with Soft Information”，*Journal of Law，Economics and Organization*，15：434—440。
6. Bolton, P. and M. Dewatripont (1994): “The Firm as a communication Network”, *Quarterly Journal Economics,* 109, 809-839.
7. Crawford, Vincent, P., and J. Sobel (1982): “Strategic Information Transmission”, *Econometrica*, 50, 1431-1451.
8. Dessein, W. (2001): “Authority and Communication in Organizations.” Manuscript, University of Chicago.
9. Faure-Grimaud, Antoine, J. J. Laffont and D. Martimort (2002): “Collusion, Delegation, and Supervision with Soft Information”, working paper, London School of Economics and University of Toulouse.
10. Gibbard, A. (1973): “Manipulation of Voting Schemes: A General Result”, *Econometrica*, 41, 587-601.
11. Gromb, Denis and D. Martimort (2002): “The Organization of Delegated Expertise”, working paper, London Business School and University of Toulouse.
12. Hart, Oliver and John Moore (1999): “On the Design of Hierarchies: Coordination Versus Specialization”, NBER working paper 7388, Cambridge, Massachusetts, USA.
13. Holmstrom， B.1979，“Moral Hazard and Observation，” *Bell Journal of Economics*，10：74—91。
14. Holmstrom, B. (1984): “On the Theory of Delegation”, in Boyer, M. and R. E. Kihlstrom ed Bayesian Models in Economic Theory, Elsevier Science Publisher

B.V..

15. Holmstrom,B., and P., Milgrom(1991): “Multitask Principal-Agent Analyses: Incentive Contracts, Assets Ownership, and Job Design,” *Journal of Law, Economics and Organization,* 7:24-51.
16. ---------- and ----------(1994): “The Firm as Incentives System,” *American Economic Review,* 84:972-991.
17. Holmstrom, B. and John Robers (1998): “The Boundaries of the Firm Revisited”, *Journal of Economic Perspectives,* 12, 73-94.
18. Kreps, David (1990): “Cooperate Culture and Economic Theory”, In Alt, J. and K. Shepsle, ed., Positive Perspectives on Political Economy, Cambridge, Cambridge university Press, 90-143.
19. Kofman, F. and J. Lawarree (1993): “Collusion in Hierarchical Agency”, *Econometrica,* 61, 629-656.
20. --------- and ---------- 1996, “On the Optimality of Allowing Collusion,” *Journal of Public Economics,*61:383—407.
21. Laffont，J.J. and D.Martimort，1997，“Collusion under Asymmetric Information，” *Econometrica*，65：875--911
22. Laffont, J. J., and D. Martimort (1998): “Collusion and Delegation”, *Rand Journal of Economics,* 29, 280-305.
23. Laffont, J. J., and D. Martimort (2000): “Mechanism Design with Collusion and Correlation”, *Econometrica,* 68, 309-342.
24. Li, Hao, and W. Suen (2001): “Delegation Decisions to Experts”, works paper, the University of Toronto and The University of Hong Kong.
25. Li, Hao, Rosen, S. and W. Suen (2002): “Conflicts and Common Interests in Committees.” *American Economic Review,* 91, 1478-1497.
26. Li, Shuhe and Weiying Zhang: (2001): “Optimal Assignment of principalship in Teams”, *Journal of Economic Behavior and Organization*, 44, 105-127.
27. Maskin, Eric and Jean Tirole (1990): “The PRINCIPAL-Agent Relationship with an Informed Principal: The Case of Private Values”, *Econometrica,* 58, 379-409.
28. Milgrom, Paul and John Roberts (1992): Economics, Organization and

Management, A Simon & Schuster Company, Englewood Cliffs, NJ, USA.

29. Morris, Stephen (1999): “Political Correctness”, Cowles Foundation working Paper, Yale University.
30. Myerson, Roger B. (1979): “Incentive Compatibility and Bargaining Problems”, *Econometrica*, 47, 61-74.
31. Myerson, Roger B. (1983): “Mechanism Design by an Informed PRINCIPAL”, *Econometrica*, 51, 1767-1797.
32. Myerson, Roger B. (1989): “Credible Negotiation Statements and Coherent Plans”, *Journal of Economic Theory,* 48, 264-303.
33. Myerson, Roger B.(1991): Game Theory: Analysis of Conflict, Cambridge, Mass.: Harvard University Press.
34. Sah, R. K. and J. Stiglitz (1986): “the Architecture of Economic Systems: Hierachies and Polyarchies”, *American Economic Review*, 76, 716-727.
35. Sah, R. K. and. Stiglit (1988): “Committees, Hierarchies, and Oligarchies”, The *Economic Journal*, 98, 451-470.
36. S. J., Grossman and Hart, O (1986): “The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration”, *the Journal of Political Economy,* 94, 691-719.
37. Segal, Ilya, (2001): “Communication Complexity and Coordination by Authority”, working paper 94720, the department of economics, UC at Berkeley.
38. Tirole, J. (1986): “Hierarchies and Bureaucracies: on the Role of Collusion in Organizations”, J*ournal of Law, Economics and Organization*, 181-213.
39. Tirole, J. (1992): “Collusion and the Theory of Organizations,” In J. J. Laffont, ed., Advances in Economic Theory, Vol. 2. Cambridge: Cambridge University Press.