Tying in Two-Sided Markets

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Abstract

Tying is widespread in two-sided markets, though it is not obvious why this should be the case. This paper offers an explanation. We extend the standard Hotelling model by allowing a duopoly to serve two distinct groups of consumers who generate externalities upon each other. We find that a quantity spillover across the two sides induces a fundamental change of strategic effects: In the presence of a large externality, price competition could lead to prices being strategic substitutes rather than strategic complements as in traditional markets. Consequently, tying works as a commitment to behave aggressively and will unambiguously hurt rivals but could be self-benefiting. Therefore, tying is adopted no matter whether a firm's aim is to deter or to accommodate rivals. Our analysis also shows that, in a duopoly, firms may engage in "prisoners' dilemma" tying. From a social planner's point of view, tying may be desirable.

Keywords: Two-sided markets, Strategic complement, Strategic substitute, Tying

JEL Classifications: L13, L14, L41

1 Introduction

This paper examines the reason why tying is widespread in two-sided markets and its impact on social welfare. Two-sided markets refer to those in which firms operate as platforms that allow interactions between two distinct groups of customers who need each other. The defining characteristic of these markets is inter-group network externalities:

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It is more valuable for consumers on one side (or less valuable in presence of negative externalities) as more consumers get on the other side. Consequently, the pricing strategy on one side of an intermediary must account for the spillover effect on the other side. Two-sided platforms arise in many economically significant industries. For instance, credit cards provide a convenient method of transaction between consumers and merchants; computer operating systems court both users and application developers; portals, TV channels, newspaper and magazines bring advertisements from producers to potential consumers; auction sites, shopping malls and real estates attempt to match sellers and buyers, etc.[1]

It is worth noting that tying is deployed in a wide variety of two-sided markets. One form of tying corresponds to the practice of bundling together two complementary goods. Consider the most famous example: Windows Media Player (WMP) is sold together with Windows. Choi (2007) analyzes this example and argues that the impact of tying on social welfare depends crucially on whether operating system users are exclusive on one media player or not. However, another form, illustrated by selling two completely independent goods together, arises even more frequently. For example, web portals offer a large bundle of services for free, such as email, web photo album, chatting tool, blog and personal website, which could as well be sold separately. Another familiar example is that CDs or make-ups are often offered together with magazines. This second kind of tying has so far received rare attentions. A major objective of our paper is to fill this gap.

Before analyzing tying of independent goods in the two-sided context, let's recall the conventional wisdom points. Whinston (1990) argues that tying acts as a commitment to lower the opportunity cost of selling the competitive good. In the pricing game that follows, the firm practicing tying will price more aggressively and as a result both firms suffer losses under tying arrangement. Hence, tying will not be adopted unless it helps drive the rivals out of the market. It is of our interest to investigate whether this result still holds in two-sided markets. Furthermore, according to Whinston (1990), tying could be welfare-reducing. It is especially true when there is little market expansion subseqent to lower prices. This result justifies many economic policies that discourage tying on the grounds of "anti-competition" or "welfare-reducing". However, the logic in one-sided markets may not work in two-sided markets (see Evans (2003), Wright (2003)). The welfare consequence of allowing tying in two-sided markets is therefore another issue that we will address here.

Although our model can be applied to a large range of two-sided industries, for con-

[1] More examples can be found in Armstrong (2006), Evans (2003) and Rochet-Tirole (2003a).

venient explanation, we will relate our model to the example of magazine industry. The tying strategy is studied in the context of Hotelling model, where two horizontally different magazines compete for single-homing readers. We characterize two-sidedness by allowing duopoly to serve another group of consumers — advertisers — whose valuation of a magazine increases in its readership. The advertisers are allowed to be multi-homing so as to reap the maximal network effect. In the basic model, we assume that readers are indifferent of the size of advertisements. We find that the externalities generated by readers on advertisers may induce fundamentally strategic changes to duopoly: contrary to the case in one-sided markets, prices could be strategic substitutes when inter-group externalities are large enough. This is due to the fact that each platform sets price of its magazine by taking into account the effects on the advertisers' side.

To start, we allow only one magazine to tie its issues with monopolized products, CD, and analyze the impact on the competition equilibrium and social welfare. We show that, in the presence of significant externalities, tying can be a profitable strategy for the firm. Consequently, in this case, tying is adopted no matter whether the firm aims at accommodating or deterring its rival, which justifies widespread tying in two-sided markets.

We further show that tying can be welfare-enhancing. This is due to the presence of larger network effect as a result of asymmetric segmentation of the market. The analysis thus has important implications for antitrust cases and provides a caution in applying the traditional policy in two-sided markets.

We then go on to test robustness of the basic model. We consider the possibility of double-direction externalities: Readers could be ad-likers or ad-dislikers. We find that, so long as readers don't distaste advertisements too much, the results in basic model still hold. With this extension, the result applies to various two-sided industries, beyond those characterized by one-direction externalities. We finally extend the analysis to the case in which both platforms could deploy tying. It turns out that the two platforms may be involved in a "prisoners' dilemma" in tying in the two-sided market rather than sticking to the "no tying" equilibrium as in one-sided markets.

Rochet and Tirole (2003b) provide an economic analysis of the tying initiated by payment card associations such as Visa and MasterCard in which merchants who accept their credit cards were forced to also accept their debit cards. They show that in absence of tying, the interchange fee between merchants' and cardholders' banks on the debit cards is too low and tends to be too high on the credit cards as compared to the social optimum. Tying is shown to be a mechanism to rebalance the interchange fee structure and to raise social welfare.

The closest model to ours is presented by Farhi and Hagiu (2007). They show that the possibility of subsidization of one side in a two-sided market can lead to fundamentally new strategic configurations in oligopoly. They present the conditions under which a cost-reducing investment by intermediaries may be a successful entry accommodation strategy and at the same time may also raise the profits of its rival, which will never happen in one-sided markets. As pointed out by Fudenberg and Tirole (1984) and Bulow Geanakoplos and Klemperer (1985), strategic effects in one-sided markets are determined by two factors: Whether actions in the competition game are strategic complements or substitutes, and whether cost-decreasing investments decrease or increase rival's profits. However, in two-sided markets, it turns to be much more complicated because there are four prices corresponding to four supply levels that need to be considered, as compared to only two variables in traditional markets. In fact, many factors can induce fundamental changes of strategic effects: price is not necessarily strategic complement in competition, the effects of cost-reducing investments on four prices are ambiguous and platforms may earn negative margin on one side. Farhi and Hagiu (2007) emphasize the last factor and show that it is enough to make strategic effect totally different whereas, in our basic model, the first factor plays an important role: in presence of large externalities, price could be strategic substitute. As a result, tying appears to be self-serving rather than self-harming, which justifies widespread tying in two-sided markets.

As we have mentioned earlier, Choi (2007) analyzes the welfare consequence of tying two complementary goods in a model of competition between two-sided platforms, where one or both sides can multi-home. In his model, tying simply allows one of the platforms to reach all consumers by bundling the platform product in question with another product that all consumers need (the motivating example is the tying of Windows Media Player to the Windows Operating System, which every PC user needs). The impact of tying on social welfare depends on whether consumers can multi-home or not, but in all cases, tying unambiguously hurts the rival platform.

Amelio and Jullien (2007) consider a setting in which two-sided platforms would like to set prices below zero on one side of the market in order to solve the demand coordination problem, but are constrained to set non-negative prices. Tying can then serve as a mechanism to introduce implicit subsidies on one side of the market in order to solve the aforementioned coordination failure. As a result, tying can raise participation on both sides and can benefit consumers in the case of a monopoly platform. In a duopoly context tying also has a strategic effect on competition. But contrary to the monopoly case, tying may not be ex-post and/or ex-ante optimal for a contested platform. Moreover, the competing platforms benefit from tying if the equilibrium implicit subsidy is large enough. We also obtain this result in the present study, although as a particular case of a broader setting. In our paper, we assume that marginal cost of magazine is large enough to avoid negative pricing and then exclude the possibility that the tied goods act as a subsidy to readers.

The remainder of the paper is organized as follows. In Section 2 we set up a two-sided duopolistic framework. Section 3 explores the competition equilibrium without tying. We analyze the effects of tying on market outcome in Section 4. Section 5 derives the welfare analysis and policy implication. In Section 6, we extend the analysis by setting that there are double-direction externalities and both firms are allowed to implement tying. Concluding remarks follows.

2 Basic Model

Platforms: Magazines

Suppose two magazines, indexed by $i = A, B$, compete for market share within readers (side 1) and advertisers (side 2). Let q_i and p_i denote prices charged readers and advertisers respectively. Production in magazine market involves no fixed cost but incurs an expenditure of c per magazine. The cost of serving advertisers side d is neglected.[2] The number of readers and advertisers who participate in platform i are denoted by n_i and m_i. We consider a situation in which at least one side is characterized by exclusive intermediation. More specifically, we assume that readers engage in single-homing while advertisers can participate in multiple platforms in order to reap maximal network benefits.

Using a standard Hotelling model: readers are heterogeneous and the number is normalized to one. They locate uniformly on a line with length equal to 1. The unit transportation cost on each side is assumed to be t. Platform A and B lie respectively on $x = 0$ and $x = 1$.

[2]The assumption that $d = 0$ is not essential in our model. The fact that $d > 0$ only change the intercept of the best reaction function of two firms. Comparing best reaction functions of two platforms without and with tying.

Without tying, the best reaction function is:

$$R_i(q_j) = \frac{4t-\alpha^2}{8t-\alpha^2} q_j + \frac{4t(t+d+\frac{2\alpha c-\alpha^2}{4})}{8t-\alpha^2} = \gamma q_j + \delta + \frac{2t\alpha \cdot d}{8t-\alpha^2}.$$

With tying, the best response system will be:

$$\begin{aligned} R_A(q_B) = &\ \gamma q_B + \delta + \tfrac{2\alpha \cdot d}{8t-\alpha^2} - \tfrac{4t \cdot s}{8t-\alpha^2}, \\ R_B(q_A) = &\ \gamma q_A + \delta + \tfrac{2t\alpha \cdot d}{8t-\alpha^2}. \end{aligned}$$

Side 1: Readers

Readers are single-homing, that is, they purchase at most one magazine. For time being, suppose that the reader side is indifferent of the size of advertisements. It can be justified on the ground that the readers come mainly for the "content". The intrinsic values of "content" of two magazines are symmetric, equivalent to v, which is assumed to be high enough that the market is totally covered. The reader locating at point x derives utility of $v - q_A - tx$ from magazine A while the net benefit of purchasing B is given by $v - q_B - t(1-x)$. Then the demand for magazine i is as follows

$$n_i = \frac{1}{2} + \frac{q_j - q_i}{2t} \ (i = A, B).$$

Side 2: Advertisers

The characteristic of two-sided market is captured by the assumption that the advertisers' willingness to pay for one magazine depends positively on its readership. More precisely, each advertiser gains additional utility of $\alpha > 0$ from each reader who reads the magazine. The net benefit of advertisers on platform i is given by $U_2^i(x) = \alpha n_i - p_i - \theta$, where θ denotes the cost of placing advertisement. Advertisers differ in θ and θ is subject to uniformly distributed in $[0, 1]$. Suppose that advertisers are allowed to be multi-homing. A θ-type advertiser will participate in platform i as long as $U_2^i(x) \geq 0$. It implies that the decision of participating relies only on the utility enjoyed on the platform, independent of that derived from the other one, which captures the fact that two magazines are not direct competitors on advertiser side and each has monopoly power. The size of advertisers is normalized to 1.Thus, the number of advertisers on platform i is given by:

$$m_i = \alpha n_i - p_i, \ (i = A, B).$$

Tying Good

To analyze the effect of tying on competition equilibrium in two-sided markets, I assume that intermediary A is also a monopolist in CD market. The production cost of CD is normalized to 0. Readers each desire at most one unit of CD. All of them have identical reservation value of $s > 0$ for CD. Platform A could sell magazines and CDs on a stand alone base or in a package.

We consider a three-stage game. In stage one, firm A determines whether or not to tie magazine and CD. The decision will be observed by firm B. In stage two, two platforms pick prices of magazines simultaneously and competition for readers takes place. In stage three, they simultaneously set prices of placing advertisements.

To solve for the equilibrium of this model, we proceed by backward induction. We firstly derive competition equilibrium taking the decision in first stage as given. The best strategy is explored by comparing the outcomes in two arrangements.

3 Platform Competition without Tying

We start with exploring competition equilibrium in absence of tying. This analysis will be used as a benchmark to investigate the effects of tying in two-sided market.

If platform A determines to sell magazine and CD separately, it extracts entire surplus and earns s by selling CDs to all readers. On the magazine market, the profit depends on sales of magazines and demand for advertising spaces. Two magazines compete in prices to attract consumers on each side sequentially. Platform i's objective function is given by:

$$\begin{aligned} \underset{p_i,\, q_i}{Max}\ \pi_i &= (q_i - c)n_i + p_i m_i \\ &= (q_i - c)n_i + p_i(\alpha n_i - p_i). \end{aligned}$$

By backward induction, platforms set advertising prices taking the magazines prices as given. Since advertisers are allowed to multi-home, each platform operates as a monopoly on advertiser side. The first order condition with respect with p_i yields the monopoly price[3]:

$$\frac{\partial \pi_i}{\partial p_i} = 0 \Rightarrow p_i = \frac{1}{2}\alpha n_i, \tag{1}$$

with corresponding profit equal to $\frac{\alpha^2}{4}n_i^2$. The advertising price p_i, independent of p_j, varies positively with readership. Advertisers place ads on a magazine for reaching prospective consumers. Naturally, the higher the level of exposure is, the more they are willing to pay for being present on the magazine and resultantly, the more ad revenue would be earned by the magazine. The total profit is therefore a function of magazine prices $\pi_i = (q_i - c)n_i(q_i, q_j) + \frac{\alpha^2}{4}n_i^2(q_i, q_j)$.

Platforms set magazine prices taking ad prices as given. First order condition with respect to q_i yields

$$n_i + (q_i - c)\frac{\partial n_i}{\partial q_i} + \frac{\alpha^2}{2}n_i\frac{\partial n_i}{\partial q_i} = 0. \tag{2}$$

[3] For example, suppose that the demand of market is $D(p) = a - p$ and the marginal cost 0. As a monopoly, he will set the monopoly price $p^m = \frac{a}{2}$. One thing different in this setting is that the willingness to pay relies on n_i.

The first two terms of LHS are as usual in the Hotelling model: To maximize profits, firms set a price by equalizing marginal revenue of selling magazines to the corresponding marginal cost. It is worth noticing the last term: a slight increase in magazine price not only influences profit on reader side but also makes it less attractive to advertisers. The fact that a smaller number of potential consumers are exposed to ads discourages the demand of advertisers and resultantly leads to a decline in profit. This additional negative effect forces platforms to set a lower price relative to one-sided markets. From (2), the best response function of platform i can be written as:

$$R_i(q_j) = \gamma q_j + \delta, \tag{3}$$

where $\gamma = \frac{4t-\alpha^2}{8t-\alpha^2}$ and $\delta = \frac{4t(t+c-\frac{\alpha^2}{4})}{8t-\alpha^2}$. The system of best reaction functions yields a symmetric equilibrium:

$$q_i^* = t + c - \frac{\alpha^2}{4}, \ (i = A, B).$$

The magazine price can be interpreted as the standard Hotelling outcome $t + c$ adjusted downward by the externality term $\frac{\alpha^2}{4}$. In equilibrium, two firms split the magazine market and charge the same price $p_i = \frac{1}{4}\alpha$ for placing ads. The total profit of each firm amounts to:

$$\pi_A^* = \frac{t}{2} - \frac{\alpha^2}{16} + s, \ \pi_B^* = \frac{t}{2} - \frac{\alpha^2}{16}.$$

We need assumptions to ensure the existence of a unique and stable equilibrium. First, for avoiding adverse selection and opportunistic behavior of agents that platforms may be confronted with by offering a direct monetary transfer to consumers, we suppose that the marginal cost of magazine is large enough that the equilibrium prices are non-negative. By doing this, we also differ our model from Amelio and Jullien (2007) in which tying goods acts as a subsidy on one side to solve the coordination problem.

Assumption 1: $c \geq \frac{\alpha^2}{4} - t$.

Since both platforms will employ more aggressive strategy in presence of externality across two groups of consumers, they earn less profit relative to one-sided case. In equation (5), we find that the larger benefits generated by readers on advertisers, the less the readers are supposed to pay for the magazine. When the externality is too large or the market competition is too vigorous, the platforms earn negative profits. In order to ensure that they are active, it must be satisfied that $t \geq \frac{\alpha^2}{8}$. As long as the industry exhibits inter-group externality, it is always true that $\gamma < \frac{1}{2}$ and $\delta > 0$. Compared with $\gamma = \frac{1}{2}$ in standard Hotelling model, the presence of advertising biases the firms towards adopting more aggressive strategy. For ruling out unstable equilibrium, we further need:

Assumption 2: $t > \frac{\alpha^2}{6}$.

It implies that $\frac{1}{2} > \gamma > -1$.

We will see in following that when $\frac{\alpha^2}{t}$ is small, the best reaction of platforms is very similar to that in one-sided markets. However, when $\frac{\alpha^2}{t}$ is large, it will be dramatically different.

3.1 Strategic Complements

It is straightforward obtained that as $\frac{\alpha^2}{t} \leq 4$, the slope of best response functions is positive, particularly $\gamma \in [0, \frac{1}{2})$, suggesting that prices charged to readers are strategic complements. This result corresponds to that in traditional markets except that firms behave more aggressively in the presence of externalities. It is clearer in figure 1, in which point curves describe the best response functions in one-sided market while solid ones represent those in two-sided market[4].

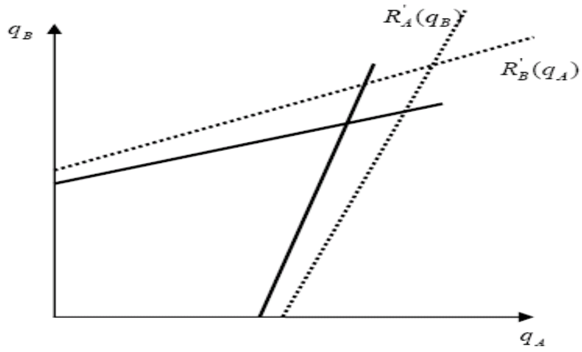

Figure 1

The interpretation of this result is that when readers generate small externalities towards advertisers, prices are strategic complements in the sense that one magazine sets a higher price in response to an increase in rival's price and vice versa. This is because one magazine tends to be more popular when its rival turns to be more aggressive. This tendency plays two conflicting roles. First, it tends to increase magazine price which brings in higher revenue with more readers in hand. It explains why prices are always strategic complements in one-sided markets. The second effect, however, arises in two-sided markets: a larger readership makes it more profitable to attract more advertisers by cutting magazine price and grabbing more readers.

[4]As a matter of fact, it may be true that $\delta > \frac{1}{2}(t + c)$ when $c > t$. However, that $\gamma < \frac{1}{2}$ is always true.

When advertisers care about readership but not so much, precisely $\frac{\alpha^2}{t} \leq 4$, the first effect dominates. Firms follow their competitor's behavior as usual in one-sided markets. The only difference is that the further consideration of advertising revenue forces them to set lower prices. It is worth mentioning that when $\frac{\alpha^2}{t} = 4$, there is no strategic interaction at all and each firm determines its own price without taking the competitor's action into account.

We find that $q_i = c + t - \frac{\alpha^2}{4} \in (c, c+t)$. Firms set magazine prices above the marginal cost and make money on both sides.

3.2 Strategic Substitutes

When the quantity spillover is significant, strategic effect changes radically. When $\frac{\alpha^2}{t} \in (4, 6)$, we have that $\gamma \in (-1, 0)$. In the presence of significant externalities, magaznine prices are instead strategic substitutes, namely as one magazine increases its price, the other reacts by reducing its price. We show the best response curves in figure 2.

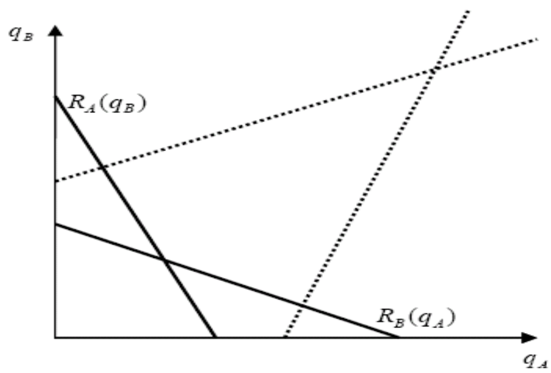

Figure 2

Proposition 1 *When $\frac{\alpha^2}{t} \leq 4$, magazine prices are strategic complements. When $\frac{\alpha^2}{t} \in (4, 6)$, prices turns out to be strategic substitutes.*

When its rival raises magazine price, a platform faces less vigorous competition, tending to charge more for magazines. This tendency is offset when advertisers care much about readership. This is because advertisers would like to pay much more for reaching one more reader when they to a larger extent mind the size of readers they are exposed to. Eventually, it turns out to be more profitable to attract more readers by cutting prices in face of a less aggressive rival.

Note that, in this case, $q_i = c + t - \frac{\alpha^2}{4} < c$. The quantity spillover is so large that in equilibrium platforms will set a price below cost to attract readers. As a result, they both make a loss on reader side, which is expected to be recouped on advertiser side. This situation is very likely to occur in practice: in a great number of industries featured by two-sidedness, such as medias, shopping malls and date clubs, platforms operate as loss-leaders on one side and make money on the other side.

4 Platform Competition with Tying

In context of one-sided markets, tying of two independent goods, though, could hurt rival, it prove to be self-defeating as well (Whinston 1990). Applying to our model, with $\alpha = 0$, platform A will never sell magazine and CD as a bundling. The question is addressed here: will it make a difference when $\alpha > 0$?

Denote $\tilde{q}_A$ the price charging for the package of one CD and one magazine A. The effective price of magazine A turns out to be $q_A = \tilde{q}_A - s$. Reader locating on x will determine to buy magazine A with a CD or only B by comparing $v + s - \tilde{q}_A - tx$ with $v - q_B - t(1 - x)$. The realized demands for A and B are respectively:

$$\begin{aligned} n_A &= \frac{1}{2} + \frac{q_B - (\tilde{q}_A - s)}{2t}, \\ n_B &= \frac{1}{2} + \frac{(\tilde{q}_A - s) - q_B}{2t}. \end{aligned}$$

The tying firm aims to maximize its prorfit composing of earnings from selling packages and advertisements

$$\underset{p_A, q_A}{Max}\ \pi_A = (\tilde{q}_A - c)n_A + p_A m_A = (q_A + s - c)n_A + p_A m_A,$$

which can also be regarded as a function of the effective price of magazine A. The determination of p_A is analogous to no tying case, the price charging for advertising increases in readership, $p_A = \frac{1}{2}\alpha n_A$. Replacing p_A in the first order condition with respect with q_A yields:

$$\frac{\partial \pi_A}{\partial q_A} = n_A + [q_A - (c - s) + \frac{\alpha^2}{2} n_A](-\frac{1}{2t}) = 0.$$

Notice that tying arrangement has the same strategic effect as a reduction in marginal cost of producing magazines from c to $c - s$. It is due to the fact that, under tying, in order to make profitable sales of CD, it must also make sales of magazine. In other word, if firm A fails to attract one reader, it also loses the opportunity to make sale of profitable CD. In the pricing competition that follows, platform A will behave more aggressively in an effort to steal readers away from firm B. On the other hand, platform B's reaction function remains unchanged. We derive the system of best response functions:

$$\begin{aligned} R_A^T(q_B) &= \gamma q_B + \delta - \frac{4t}{8t-\alpha^2}s, & (4)\\ R_B^T(q_A) &= \gamma q_A + \delta. & (5)\end{aligned}$$

By solving (4), (5), we obtain competition equilibrium under tying.

$$q_A^T = q_A^* - \frac{8t-\alpha^2}{2(6t-\alpha^2)}s, \quad q_B^T = q_B^* - \frac{4t-\alpha^2}{2(6t-\alpha^2)}s.$$

Not surprising, firm A sets a lower effective price of magazine compared with separate selling. On the other hand, the reaction of magazine B to tying is ambiguous. It may raise or reduce magazine price depending on the extent of externalities. One interesting result is that when advertisers care so much about readers that satisfying $\frac{\alpha^2}{t} \in (4,6)$, magazine B turns out to be more expensive under tying. The equilibrium market share under tying is that

$$n_A^T = \frac{1}{2} + \frac{s}{6t-\alpha^2}, \ n_B^T = \frac{1}{2} - \frac{s}{6t-\alpha^2}.$$

Firm A definitely grabs a larger market share by selling CD and magazine in a package. The sales of CD decline to n_A^T from 1. Magazine B will lose some readers even though it may react to tying by behaving aggressively. The adverting price and demand are both linear in readership. It is clear that firm A is expected to receive more advertising revenue while its rival faces a reduction in profit from advertiser side. Finally, they respectively earn profit as follows:

$$\begin{aligned} \pi_A^T &= \pi_A^* + \frac{s[(8t-\alpha^2)s - (6t-\alpha^2)(16t-3\alpha^2)]}{4(6t-\alpha^2)^2},\\ \pi_B^T &= \pi_B^* - \frac{s(8t-\alpha^2)(6t-\alpha^2-s)}{4(6t-\alpha^2)^2}.\end{aligned}$$

In the rest of this section, we will discuss the impact of tying on profits by examining the variation of profits with the value of tying good. Under separating selling, CD's value has no effect on magazine B's profit while the earnings of firm A increases in s at a rate of 1. Conversely, the value of CD is critical when it is tying with magazine A as a package to compete against magazine B. Tying would be a revenue-increasing strategy if and only if A's profit increases in s at a rate larger than 1, which could be true only when prices are strategic substitutes.[5]

[5] This is the necessary but not sufficient condition that tying is a profitable strategy.

4.1 Strategic Complements

When $\frac{\alpha^2}{t} \leq 4$, prices are strategic complements. Tying arrangement shifts firm A's reaction curve leftwards. The change on the equilibrium can be seen in Figure 3.

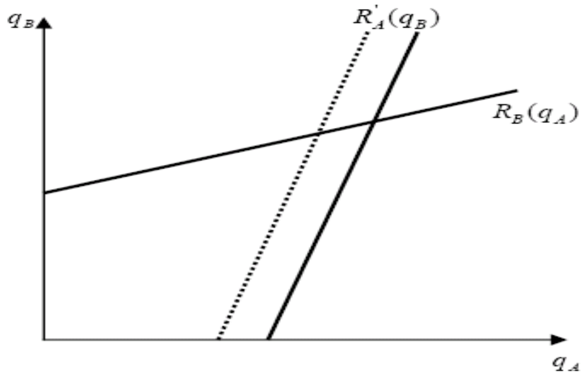

Figure 3

In equilibrium, both platforms cut magazine prices. The impact of tying on magazine B's profit can be identified by comparing the differentiation of π_B with respect to s under tying and no tying. By Envelope Theory, the only influence of tying is through strategic effect.[6]

$$\frac{d\pi_B}{ds} = \underbrace{\frac{\partial \pi_B}{\partial s} + \frac{\partial \pi_B}{\partial q_B} \cdot \frac{dq_A^T}{ds}}_{=0} + \underbrace{\frac{\partial \pi_B}{\partial q_A} \cdot \frac{dq_A^T}{ds}}_{Strategic\ Effect} = \underbrace{n_B^T}_{>0} \cdot \underbrace{\frac{dq_A^T}{ds}}_{<0} < 0.$$

Under tying, the higher the value of tying good is, the less profit platform B will make. As compared with no tying case, it is obvious that platform B suffers loss from tying. Firm A charges unambiguously less for magazines and takes readers away from B. Thereby, after tying, platform B serves less consumers at a lower price on both sides.

Concerning firm A, direct effect and strategic effect comprise influences of tying on A's profit:

$$\frac{d\pi_A}{ds} = \underbrace{\frac{\partial \pi_A}{\partial s}}_{Direct\ Effect} + \underbrace{\frac{\partial \pi_A}{\partial q_B}\frac{dq_B^T}{ds}}_{Strategic\ Effect} = \underbrace{n_A^T}_{\leq 1} + \underbrace{n_A^T}_{>0} \cdot \underbrace{\frac{dq_B^T}{ds}}_{<0} < 1.$$

[6]The strategic effect is

$$\begin{aligned}\frac{\partial \pi_B}{\partial q_A} \cdot \frac{dq_A^T}{ds} &= (\frac{\alpha^2}{2} n_B + q_B - c)\frac{\partial n_B}{\partial q_A}\frac{dq_A^T}{ds} \\ &= n_B^T \frac{dq_A^T}{ds},\end{aligned}$$

since first order condition yields $\frac{\partial \pi_B}{\partial q_B} = (\frac{\alpha^2}{2} n_B + q_B - c)\frac{\partial n_B}{\partial q_B} + n_B = 0$.

The direct effect, $\frac{\partial \pi_A}{\partial s} \leq 1$, captures the fact that firm A loses money on CD market. Under no tying, A's profit increases in value of CD at a rate of 1, the realized size of purchasers of CD. Firm A fails to extract surplus from all readers because some of them give up consuming CDs under tying. The strategic effect is negative due to the fact that tying arrangement forces firm B to behave more aggressively, leading magazine market to be more competitive. It makes firm A less profitable in magazine market compared with independent pricing case.

Proposition 2 *When* $\frac{\alpha^2}{t} \leq 4$*, both platforms earn less under tying arrangement.*

The similar result is obtained in one-sided markets and no tying is an optimal strategy in this context.[7] According to analysis above, this result survives in two-sided markets so long as prices are strategic complements.

4.2 Strategic Substitutes

When inter-group externality is so large, $\frac{\alpha^2}{t} \in (4, 6)$, that satisfies $-1 < \gamma < 0$, magazine prices being strategic substitutes. Platform A still adopts more aggressive strategy for making more sales of monopolized good. However, platform B reacts by raising magazine price. This is because when externalities across two groups are significant, a smaller readership under tying makes it much less costly for magazine B to set a high price for advertising. Most importantly, this effect offsets the tendency to decrease price in face of more vigorous competition caused by lower q_A. The equilibrium is illustrated in figure 4: the effective price of magazine A declines whereas that of magazine B rises. In all cases, tying works as a creditable commitment to set a low price. It will force the rival increase its price when prices are strategic substitutes.

[7] See Whinston (1990). In the context of one-sided market, one firm will choose tying unless it drives rival's profit to negative and drive it out of the market. However, firm B never earn negative profit in our setting since production incurs no fixed cost.

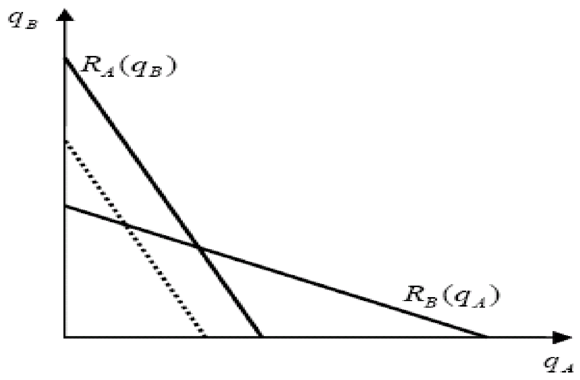

Figure 4

We now analyze the effect of tying on magazine market. As analyzed above, tying is a "tough" strategy in the sense that it makes firm B less profitable.

$$\frac{d\pi_B^T}{ds} = \underbrace{n_B^T}_{>0} \underbrace{\frac{dq_A^T}{ds}}_{<0} < 0.$$

This is because no matter whether the prices are strategic complements or substitutes, A decreases q_A under tying. However, the impact on platform A's profit may be radically different. When A behaves more aggressively, q_B^T rises. The fact that prices could be strategic substitutes in context of two-sided markets opens new possibilities: firm A may benefit from practicing tying.[8]

$$\frac{d\pi_A^T}{ds} = \frac{\partial \pi_A}{\partial s} + \frac{\partial \pi_A}{\partial q_B} \cdot \frac{dq_B^T}{ds} = n_A^T \cdot \underbrace{[1 + \frac{dq_B^T}{ds}]}_{>1}.$$

When $\frac{d\pi_A}{ds} > 1$, it may be true that selling magazine and CD in a package is more profitable. Since less readers will purchase CDs, losing money in CD market is inevitable. But this part of losses may be compensated because platform A may earn more money in magazine market, which is impossible in one-sided context. The intuition behind

[8] Note that $\frac{\partial \pi_A}{\partial s} = n_A^* \leq 1$ and

$$\begin{aligned} \frac{\partial \pi_A}{\partial q_B} \cdot \frac{dq_B^T}{ds} &= (\frac{\alpha^2}{2} n_A + q_A - c) \frac{\partial n_A}{\partial q_B} \frac{dq_B^T}{ds} \\ &= n_A^T \frac{dq_B^T}{ds}, \end{aligned}$$

since first order condition yields $\frac{\partial \pi_A}{\partial q_A} = (\frac{\alpha^2}{2} n_A + q_A - c+) \frac{\partial n_A}{\partial q_A} + n_A = 0$.

this result is that tying arrangement forces platform B to pick a higher price, which in turns brings to the tying firm a much larger share of readers and corresponding far more advertising revenue.

Furthermore, notice that the price of package $\tilde{q}_A^T = q_A^T + s = t + c - \frac{\alpha^2}{4} + \frac{4t-\alpha^2}{2(6t-\alpha^2)}s$, a price even lower than that charged for magazine A under separate selling. Under tying, platform A affords more losses on reader side while B reduces losses due to a smaller market share and less loss from each reader. Although B "derives" more from readers, it loses much more on advertiser side. On the other hand, A's losses on reader side will be compensated.

Proposition 3 *When $\frac{\alpha^2}{t} \in (4,6)$, platform B definitely suffers losses while platform A may benefit from tying.*

Platform A may choose tying which will never occur in absence of externalities. This result justifies prevalence of tying in two-sided markets. In the following, we will identify conditions under which the extra benefits on magazine market offset the losses on CD market so that tying turns out to be a good strategy.

4.3 Tying or not?

Since production of magazine doesn't incur fixed cost at all, it is impossible to deter rival through tying. Then the decision of whether tying or not is driven by maximizing $\pi_{A.}$

As obtained above, when $\frac{\alpha^2}{t} \leq 4$, tying will never be a profitable strategy. However, when $\frac{\alpha^2}{t} \in (4,6)$, prices become strategic substitutes and then A may benefit from tying on magazine market. Although it continues to earn less on CD market relative to separate selling, it may make more money on magazine market, in particular, on advertiser side. The gain of firm A by tying two independent goods is:

$$\Delta\pi_A \equiv \pi_A^T - \pi_A^* = \frac{s\ (8t-\alpha^2)}{4(6t-\alpha^2)^2}[s - \frac{6t\ -\alpha^2}{8t-\alpha^2}(16t - 3\alpha^2)].$$

Firm A will choose tying if and only if extra profit on magazine market can offset losses in CD market. Precisely, $\Delta\pi_A > 0$ when $s \geq \max\{0, (16t-3\alpha^2)\frac{6t-\alpha^2}{8t-\alpha^2}\}$.

Proposition 4 *(1) When $\frac{\alpha^2}{t} \in [\frac{16}{3}, 6)$, platform A chooses tying for all $s > 0$.*

(2) When $\frac{\alpha^2}{t} \in [\frac{24}{5}, \frac{16}{3})$, firm A chooses tying if the value of CD is so large that satisfying $s > \tilde{s} \equiv (16t-3\alpha^2)\frac{6t-\alpha^2}{8t-\alpha^2}$.

(3) When $\frac{\alpha^2}{t} < \frac{24}{5}$, tying is never adopted.

Proof. *Since the market is covered and reader side is not allowed to expand in this model,* $n_A^T = \frac{1}{2} + \frac{s}{6t-\alpha^2} \le 1$*.If firm A sells the magazine with a CD of value* $s \ge \bar{s} = \frac{6t-\alpha^2}{2}$*, it holds the entire market. In other words, all the tying goods of value* $s > \bar{s}$ *have the same strategic effect. If tying is not a profitable strategy at* $s = \bar{s}$*, the tying firm will earn the same profit in magazine market by tying a good of* $s > \bar{s}$*. (In the following figure, we could see that when* $s > \bar{s}$*, A's profit increases in* s *at rate equal to* 1*, which represents the profit of CD market increases in* s *at rate equal to* 1 *while that of magazine market is constant as* $s \ge \bar{s}$*.)*

Therefore, $\tilde{s}$ *and* $\bar{s}$ *are two critical points. If and only if* $\tilde{s} \le \bar{s}$ *and* $s > \tilde{s}$*, tying is a self-benefiting strategy. Straightforward,* $\tilde{s} \le \bar{s}$ *if and only if* $\frac{\alpha^2}{t} \ge \frac{24}{5}$*.*

When $\frac{\alpha^2}{t} \in [\frac{16}{3}, 6)$*,* $\tilde{s} \le 0$*.* $\Delta\pi_A > 0$ *as long as* $s > 0 \ge \tilde{s}$*.*

When $\frac{\alpha^2}{t} \in [\frac{24}{5}, \frac{16}{3})$*,* $\tilde{s} > 0$*.* $\Delta\pi_A > 0$ *if* $s > \tilde{s} > 0$*. .*

When $\frac{\alpha^2}{t} < \frac{24}{5}$*, tying is definitely self-harming.* ■

In figure 5, we could find curves representing A's profit respectively in three cases which we mentioned in proposition.

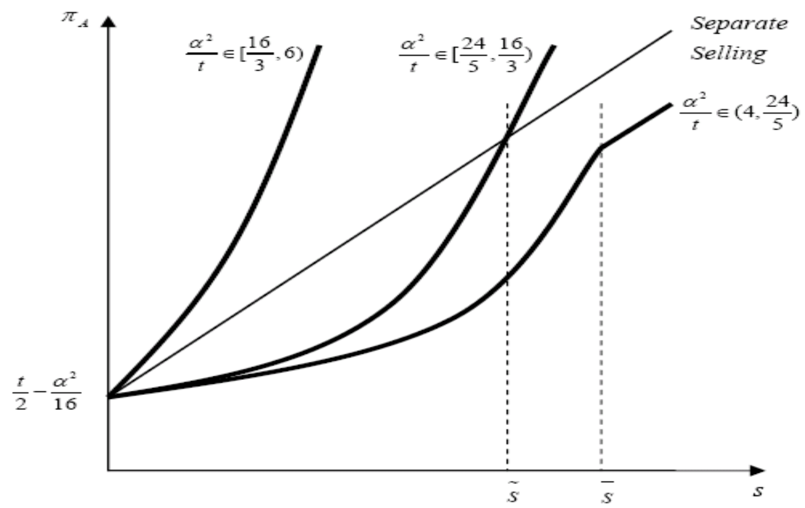

Figure 5

In case of $\frac{\alpha^2}{t} \in [\frac{16}{3}, 6)$, tying strategy dominates: A always sells magazine and CD together. A large externality or a small transportation cost implies that a small advantage will lead to a drastic change of market share of readers which brings in much more advertising revenue. The additional benefit is so large that the lost money on CD market is neglectable. When $\frac{\alpha^2}{t} \in [\frac{24}{5}, \frac{16}{3})$, firm A receives more money on magazine market at the expense of profit on CD market. Whether tying arrangement would be more profitable depends upon to which extent it sacrifices income of selling CD. Larger s implies that less consumers give up purchasing CD under tying. Platform A can finds a critical point $\tilde{s}$, above which tying proves to be more beneficial strategy. When $\frac{\alpha^2}{t} < \frac{24}{5}$, firm A is unable to improve profit by adopting tying even if it grabs all readers.

5 Welfare Analysis

In section 3 and 4, we have provided an explanation for the prevalence of tying in two-sided markets and identified conditions under which tying would be adopted. This section will contribute to welfare analysis. The question whether this kind of tying is socially desirable will be addressed.

The social welfare of this market consists of advertisers' surplus, readers' surplus and two platforms' profits. Under separate selling, each platform owns half of the magazine market.

$$\begin{aligned} W_M^* &= \underbrace{\int_0^{m_A^*}(\alpha n_A^* - x)dx + \int_0^{m*}[\alpha n_B^T - x)]dx}_{W_1} \\ &\quad \underbrace{+\int_0^{n_A^*}(v - tx - c)dx + \int_{1-n_B^*}^{1}[v - t(1-x) - c]dx}_{W_2} \\ &= v - c - \frac{t}{4} + \frac{3}{16}\alpha^2 \end{aligned}$$

where $W_i(i = 1, 2)$ denotes social welfare of side i. Social surplus of the CD market is $W_{CD} = s$. The total social welfare equals $W = v - c - \frac{t}{4} + \frac{3}{16}\alpha^2 + s$.

The variation of total surplus due to tying can be expressed as:

$$\begin{aligned} \Delta W &= \underbrace{\Delta W_1}_{<0 \ or \ >0} + \underbrace{\Delta W_2}_{<0} + \underbrace{(n_A^T - 1)s}_{<0} \\ &= [\frac{\alpha^2 s^2}{(6t-\alpha^2)^2} - \frac{3\alpha^2 s^2}{4(6t-\alpha^2)^2}] + \frac{-s^2}{(6t-\alpha^2)^2} + (n_A^* - 1)s \\ &= \frac{s \cdot (20t - \alpha^2)}{(6t-\alpha^2)^2}[s - \frac{2(6t-\alpha^2)^2}{20t-\alpha^2}]. \end{aligned} \tag{6}$$

There are three channels through which tying affects social welfare. First, since there is no market expansion on single-homing side, the reader side worsens off due to a rising transportation cost. Second, fewer readers would buy CDs. The interpretation is that some readers fail to purchase their favorite magazine because they are reluctant to give up CDs while other prefer sticking to magazine B at the expense of losing the opportunity of having CDs. They both are inefficient from the point view of social planner, which explains why tying is always welfare-reducing in one-sided market. However, the third effect arises in two-sided market. Under tying, the market is segmented into a big part and a small one. Due to the network effect brought by inter-group externalities, such an asymmetric market structure is preferred to a symmetric one with two medium-sized parts.[9] The net effect of tying on social welfare depends on three elements: the externality, transportation cost and value of tying good.

Proposition 5 *When $\frac{\alpha^2}{t} \leq \frac{4}{3}$, tying never improves social welfare. When $\frac{\alpha^2}{t} \in (\frac{4}{3}, 6)$, tying is welfare-enhancing if and only if $s > \hat{s} = \frac{2(6t-\alpha^2)^2}{20t-\alpha^2}$.*

Proof. Tying is welfare-improving when $\Delta W > 0$. Solving this inequation, we have the $s > \hat{s} = \frac{2(6t-\alpha^2)^2}{20t-\alpha^2}$.

Note that tying a magazine with a CD of value higher than $\bar{s} = \frac{6t\ -\alpha^2}{2}$ leads to the same market structure: platform A holds all readers and all advertisers. In other words, if tying doesn't improve social welfare at $s = \bar{s}$, it will in no way be true when $s > \bar{s}$. We need pose another constraint that $\hat{s} < \bar{s}$. It is easy to show that $\hat{s} < \bar{s}$ when $\frac{\alpha^2}{t} > \frac{4}{3}$. ■

Contrary to one-sided markets, tying could be welfare-enhancing when the value of tying good is high enough. As $\frac{\alpha^2}{t} \leq \frac{4}{3}$, tying would never be welfare-enhancing. As transportation cost is pretty large whereas externalities are not significant, the loss on reader side is unable to be covered even though all readers and advertisers interact on the same platform so that the maximal network effect is realized. When $\frac{\alpha^2}{t} \in (\frac{4}{3}, 6)$, we could find a region $s > \hat{s}$ in which tying is welfare-enhancing. A higher value of $\frac{\alpha^2}{t}$ indicates that a much larger network effect is realized under tying at the expense of a small increase in transportation cost. The value of tying good is relevant because it is critical in determining the market share. Tying with a CD of higher value makes the package more attractive and consequently leads to less loss on CD market and a markedly asymmetric segment of the market.

Recall that platform A would tie magazine and CD only if $\frac{\alpha^2}{t} \in [\frac{24}{5}, 6)$. There must be some circumstances under which tying is socially desirable but never adopted by firm A. We could conclude all these results in a two-dimension figure of $(\frac{\alpha^2}{t}, s)$. In figure 6, the gray region represents that in which platform A will tie two goods and tying is socially desirable. In the black region, tying is welfare-harming but profitable for platform A.

[9] The intuition parellels that of convexity: $(a^2 + b^2) \geq 2(\frac{a+b}{2})^2$.

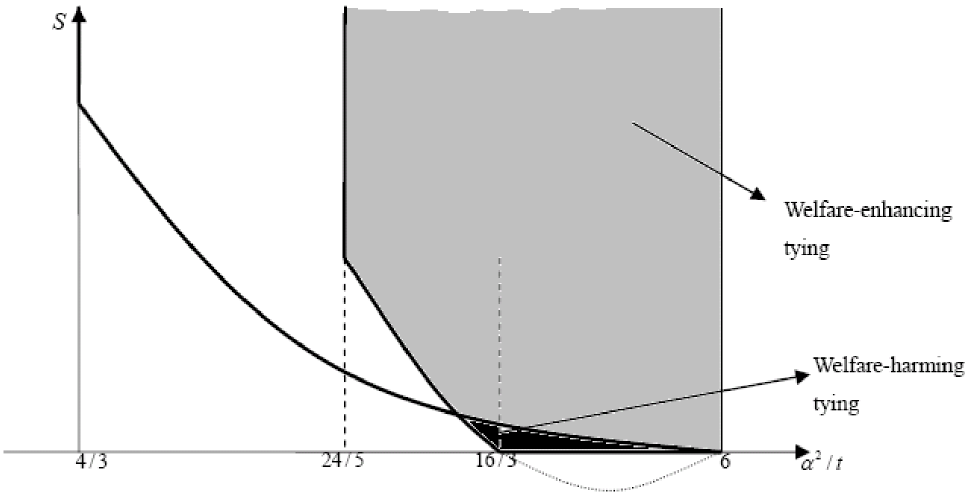

Figure 6

6 Extensions

6.1 Ad-like Readers or Ad-dislike Readers

We now generalize the basic setting: Advertisements also generate externalities β towards readers. β could be positive or negative, corresponding to ad-like and ad-dislike readers respectively. This general model can be applied to most two-sided industries rather than some special ones characterized by one-direction externality.

The number of advertiser participating in platform i remains unchanged

$$m_i = \alpha n_i - p_i \quad (i = A, B),$$

while the number of readers purchasing magazine i turns to be:

$$n_i = \frac{1}{2} + \frac{\beta(m_i - m_j)}{2t} + \frac{q_j - q_i}{2t} \quad (i = A, B).$$

For simplicity, suppose the marginal cost of magazine is $c = 0$, which will not influence the robustness of our results.[10] In the same manner as in section 3, we could derive the best response function of each platform.

[10]Since assumption 1 is no longer satisfied, we allow negative prices.

Proposition 6 *The best response function of magazine i is that*

$$R_i(q_j) = \frac{\Omega q_i + \digamma}{\Gamma}, (i = A, B),$$

where $\Gamma = \frac{256t^4 - 32t^3(\alpha^2+20\alpha\beta+\beta^2)+64t^2\alpha\beta(\alpha^2+9\alpha\beta+\beta^2)-2t\alpha^2\beta^2(21\alpha^2+110\alpha\beta+20\beta^2)+\alpha^3\beta^3(9\alpha^2+30\alpha\beta+7\beta^2)}{(2t-\alpha\beta)(4t-3\alpha\beta-\beta^2)} > 0$, $\Omega = 16t^2 - 4t\alpha(\alpha + 15\beta) + \alpha^2\beta(3\alpha + 5\beta)$ *and* $\digamma = \frac{\Omega(4t-\alpha\beta)(t-\alpha\beta)}{4t-3\alpha\beta-\beta^2}$.

Magazine prices are strategic substitutes when $t \in [\max\{0, \alpha\beta\}, \frac{1}{8}(\alpha^2+5\alpha\beta+\alpha\sqrt{\alpha^2 - 2\alpha\beta + 5\beta^2})]$.

See appendix for the proof. The implicaiton of prices being strategic substitutes is that platforms find it more attractive to seize one additional reader when they seize a larger market share. The following is a two-dimension figure of (β, t). The shadow part represents the region in which prices are strategic substitutes. Notice that, given the value of α, for all $\beta \in [-\frac{3}{5}\alpha, \alpha]$, we could always find an interval of t in which prices are strategic substitutes.

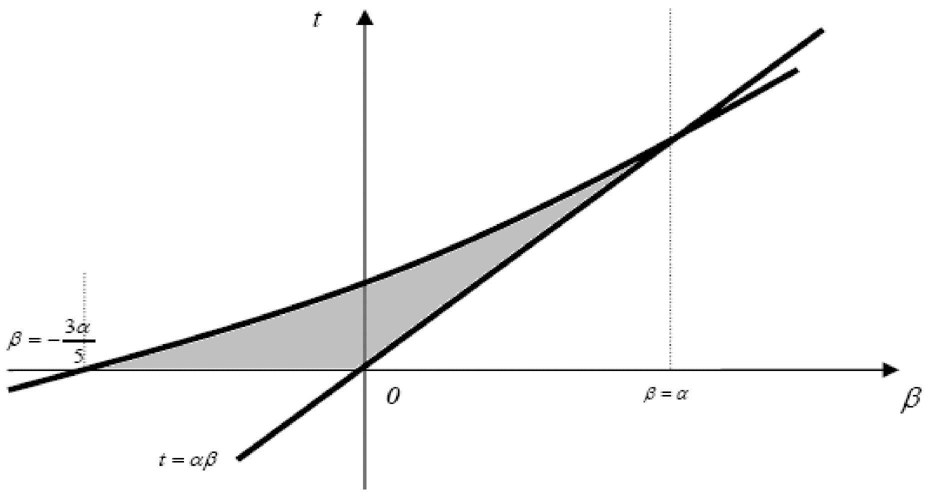

Figure 7

Solving the system of best response functions, we obtain the symmetric equilibrium

$$q_A^* = q_B^* = \frac{(2t - \alpha\beta)\Omega}{2[16t^2 - 20t\alpha\beta + \alpha^2\beta(6\alpha + \beta)]}.$$

It is not difficult to derive the new equilibrium when platform A adopts tying

$$\begin{aligned} q_A^T &= q_A^* - [1 - \frac{(2t - \alpha\beta)\Omega}{\Phi}]s, \\ q_B^T &= q_B^* - \frac{(2t - \alpha\beta)\Omega}{\Phi}s, \end{aligned}$$

where $\Phi = 2[48t^3 - 4t^2(2\alpha^2 + 21\alpha\beta + 2\beta^2) + t\alpha\beta(10\alpha^2 + 46\alpha\beta + 9\beta^2) - \alpha^2\beta^2(3\alpha^2 + 8\alpha\beta + 2\beta^2)] > 0$ to ensure that there exists a unique and stable equilibirium. Note that platform B may set a higher or lower price for magazines in response to A's tying arrangement depending upon the sign of Ω. The impact on platform A's profit of tying a CD of value s with its magazine can be expressed by

$$\frac{d\pi_A}{ds} = n_A^T + \frac{\partial \pi_A}{\partial q_B} \cdot \frac{\partial q_B^T}{\partial s}.$$

As we have analyzed in section 4, there are opportunities that tying could be a profitable strategy if and only if $\frac{\partial \pi_A}{\partial q_B} \cdot \frac{\partial q_B^T}{\partial s} > 0$.

Proposition 7 *Tying could be a profitable strategy if and only if prices are strategic substitutes.*

See proof in the appendix. Actually, we find that $\frac{\partial \pi_A}{\partial q_B} > 0$, implying that platform A will benefit from an increase in rival's price. Tying arrangement proves to force the rival to set a higher price when magazine prices are strategic substitutes. The result we have got in main part still hold in general case.

6.2 Prisoners' Dilemma

Finally, we enrich the model by allowing both firms to tie their magazines with monopolized goods. Suppose platform B monopolizes a DVD market. For simplicity, we assume that readers value DVDs at s, the same as CDs. Under separate selling, all readers would like to purchase one unit of DVD and CD as long as the prices is below s.

We give a new two-stage game: In stage one, platform A and B decides to tie or not simultaneously. The decisions are common knowledge. Price competition will happen in stage two.

Firstly, we explore the Nash-equilibrium in the absence of externalities. Given the rival selects "no tying", no one will choose "tying". This is because tying is unable to increase revenue since it leads to losses of tying good and at the same time drives the market of tied good much more competitive. Provided the rival selling two goods in a package, one firm earns $\frac{t}{2}$ by following the same strategy or $\frac{t}{2} + \frac{2s}{3} - \frac{s^2}{18}$ by keeping selling two goods separately. It is not difficult to show that "no tying" is a profitable strategy.[11] Therefore, all firms stick to "no tying" strategy for any choice of the rival. (No tying, no tying) turns out to be a Nash-equilibrium in one-sided markets.

[11] Since $n_A^* \leq 1$, $s \leq 3t$. Then $\frac{t}{2} + \frac{2s}{3} - \frac{s^2}{18} > \frac{t}{2}$.

Platform A

Platform B	$\alpha = 0$	no tying	tying
	no tying	$\boxed{\frac{t}{2}+s}, \boxed{\frac{t}{2}+s}$	$\frac{t}{2}+\frac{s}{3}+\frac{s^2}{18}, \boxed{\frac{t}{2}+\frac{2s}{3}-\frac{s^2}{18}}$
	tying	$\boxed{\frac{t}{2}+\frac{2s}{3}-\frac{s^2}{18}}, \frac{t}{2}+\frac{s}{3}+\frac{s^2}{18}$	$\frac{t}{2}, \frac{t}{2}$

Now we move to the case in which externalities arise. We have obtained the conditions under which tying arrangement proves to bring about more incomes as the rival chooses separate selling. We will continue by exploring the conditions under which "tying" is the best response to rival's "tying" strategy. It is straightforward obtained by comparing income under separate selling composed of revenue from magazines and s by selling DVDs to all readers

$$\pi_B^{NT} = \frac{(8t-\alpha^2)(6t-\alpha^2-2ts)^2}{16(6t-\alpha^2)^2} + s,$$

with income in tying case where two packages compete with each other and eventually share the market evenly

$$\pi_B^T = \frac{t}{2} - \frac{\alpha^2}{16}.$$

The best response is "tying" if it is satisfied that $s \in (0, -(16t-3\alpha^2)\frac{6t-\alpha^2}{8t-\alpha^2}]$. The intuition is that when the expected profit from tying good is not quite attractive, platforms find it better to leverage market power of tying good to magazines market through tying. This could be true if and only if $\frac{\alpha^2}{t} \in (\frac{16}{3}, 6)$ and $s \in (0, -(16t-3\alpha^2)\frac{6t-\alpha^2}{8t-\alpha^2}]$, the conditions under which "tying" is a dominant strategy according to proposition 4.

Proposition 8 *When $\frac{\alpha^2}{t} \in (\frac{16}{3}, 6)$, (tying, tying) is a Nash-equilibrium as long as the value of tying good is small enough, satisfying that $s \in (0, -(16t-3\alpha^2)\frac{6t-\alpha^2}{8t-\alpha^2}]$.*

Platform A

Platform B	$\frac{\alpha^2}{t} \in (\frac{16}{3}, 6)$	no tying	tying
	no tying	$\frac{t}{2}-\frac{\alpha^2}{16}+s, \quad \frac{t}{2}-\frac{\alpha^2}{16}+s$	$\boxed{\varphi(\chi+2s)^2}, \varphi(\chi-2s)^2+s$,
	tying	$\varphi(\chi-2s)^2+s, \boxed{\varphi(\chi+2s)^2}$	$\boxed{\frac{t}{2}-\frac{\alpha^2}{16}}, \boxed{\frac{t}{2}-\frac{\alpha^2}{16}}$

where $\varphi = \frac{8t-\alpha^2}{16(6t-\alpha^2)^2}$ and $\chi = 6t - \alpha^2$.

On the contrary, the strategy "tying" will never be selected to compete with a "tying" rival when $\frac{\alpha^2}{t} \leq \frac{16}{3}$. It is because when the externalities are not significant, the additional incomes by "stealing" readers will never be high enough to make up for the sacrifying money on market of tying good. When $\frac{\alpha^2}{t} \leq \frac{25}{4}$, "no tying" is a dominant strategy.

Platform A

	$\frac{\alpha^2}{t} \leq \frac{25}{4}$	no tying	tying
Platform B	no tying	$\boxed{\frac{t}{2} - \frac{\alpha^2}{16} + s}$, $\boxed{\frac{t}{2} - \frac{\alpha^2}{16} + s}$	$\boxed{\varphi(\chi + 2s)^2}$, $\varphi(\chi - 2s)^2 + s$
	tying	$\varphi(\chi - 2s)^2 + s$, $\boxed{\varphi(\chi + 2s)^2}$	$\frac{t}{2} - \frac{\alpha^2}{16}$, $\frac{t}{2} - \frac{\alpha^2}{16}$

Proposition 9 *When $\frac{\alpha^2}{t} \leq \frac{25}{4}$, (no tying, no tying) is a Nash-equilibrium for all $s > 0$.*

7 Conclusion

Traditional analysis of tying has focused on conventional markets. In such markets a general insight is that the firm can harm the rival by tying two independent goods, which will also reduce its own profit. Our analysis has shown that this is challenged in a two-sided market. We construct a simple model of two-sided markets, in which two magazines competes for readers as in the standard Hotelling model and, on the other hand, they serve the advertisers whose demand relies positively on the size of readers. When the externality generated by the readers on the advertisers is large enough, the prices set by the duopoly are strategic substitutes and then tying could be self-benefiting. As a result, tying will be adopted whether the firm aims at accommodating or deterring rival.

Our analysis then proceeds to examine the effects of tying on social welfare. Contrary to the conventional wisdom, tying could be welfare-enhancing in two-sided markets. It is due to the fact that, in presence of the network effect, optimal allocation of consumers on both sides should be asymmetric. In tying regime, the inefficiency in the market of magazine is mitigated. This result has important implications for competition policy in two-sided markets.

Our study has been started by a setting where only readers generate externalities on advertisers. Then we check the robustness of results in presence of two-direction externalities and the main results in this paper survive in more general setting. By doing this, we avoid incorporating any of the particularities of the media market into the model and therefore highlight the most common mechanisms of tying in two-sided markets. In the end, we extend the analysis by allowing both firms to tie the magazine with a monopolized good and argue that they may be involved in "prisoner's dilemma".

In our model, we assumed that the value of monopolized good is exogenous and we find that it plays an important role when the platform decides to tie or not. Interesting future wok might relax this assumption and analyze a model in which the platform should determine the value (or the quality) of the monopolized goods preceding the game in the basic model.

Appendix

Proof of Proposition 6. Solving $m_i = \alpha n_i - p_i$ $(i = A, B)$ and $n_i = \frac{1}{2} + \frac{\beta(m_i - m_j)}{2t} + \frac{q_j - q_i}{2t}$ $(i = A, B)$, we obtain that demand for magazines and advertising are functions of p_A, p_B, q_A and q_B.

$$\begin{aligned} m_i &= \frac{\alpha}{2} + \frac{\alpha(q_j - q_i)}{2(t - \alpha\beta)} + \frac{\alpha\beta \cdot p_j - (2t - \alpha\beta)p_i}{2(t - \alpha\beta)}, \\ n_i &= \frac{1}{2} + \frac{q_j - q_i}{2(t - \alpha\beta)} + \frac{\beta(p_j - p_i)}{2(t - \alpha\beta)}, \end{aligned}$$

where $t > \alpha\beta$. The demand for advertising of magazine i unambiguously decreases in p_i whereas increases or decreases in rival's price depending on sign of β. For instance, $\frac{\partial m_i}{\partial p_j} < 0$ when readers dislike advertisements. This is because a higher advertising price brings in fewer advertisements but makes the magazine more competitive in attracting readers which in turns hits its rival in competition for advertisers. The objective function of platform i is then as follows

$$\max_{q_i,\, p_i} \pi_i = p_i \cdot m_i(p_i, p_j, q_i, q_j) + q_i \cdot n_i(p_i, p_j, q_i, q_j).$$

By backward induction, taking p_j, q_i, q_j as given, platform i maximizes its profit by setting p_i. Note that, although magazines do not compete directly with each other for advertisers, one needs take the rival's charge for advertising into account when setting its own advertising price since the resultant demands for advertisements will affect direct competition on reader side. We have that[12]

$$p_i(p_j) = \frac{\alpha\beta}{4t - 2\alpha\beta} p_j + \frac{-(\alpha + \beta)q_i + \alpha q_j + \alpha(t - \alpha\beta)}{4t - 2\alpha\beta}, \ (i \neq j),$$

with second order condition requiring that $2t - \alpha\beta > 0$. When readers are ad-like, advertising prices are strategic complements while $p_i'(p_j) < 0$ otherwise. In equilibirium, advertising price of each magazine depends on q_A and q_B

$$p_i(q_i, q_j) = \frac{\alpha(4t - 3\alpha\beta - \beta^2)q_j - [4t(\alpha + \beta) - \alpha\beta(3\alpha + 2\beta)]q_i + \alpha(4t - \alpha\beta)(t - \alpha\beta)}{(4t - 3\alpha\beta)(4t - \alpha\beta)}.$$

[12] Second order conditions require that

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = \frac{2t - \alpha\beta}{-t + \alpha\beta} < 0 \text{ and } \frac{\partial \pi_A}{\partial p_A} \cdot \frac{\partial \pi_B}{\partial p_B} - (\frac{\partial^2 \pi_A}{\partial p_A \partial p_B})^2 = \frac{(4t - 3\alpha\beta)(4t - \alpha\beta)}{4(t - \alpha\beta)^2} > 0.$$

So we have that $2t - \alpha\beta > 0$ and $(4t - 3\alpha\beta)(4t - \alpha\beta) > 0$.

Replacing p_i by $p_i(q_i, q_j)$, the number of participants on each side are functions of q_i and q_j

$$\begin{aligned} m_i(q_i, q_j) &= \frac{\alpha(4t - 3\alpha\beta - \beta^2)q_j - [8t^2(\alpha - \beta) - 2t\alpha\beta(5\alpha - 4\beta) + \alpha^2\beta^2(3\alpha - \beta)]q_i}{2(t - \alpha\beta)(4t - 3\alpha\beta)(4t - \alpha\beta)} \\ &\quad + \frac{\alpha}{2(4t - 3\alpha\beta)}. \\ n_i(q_i, q_j) &= \frac{(4t - 3\alpha\beta - \beta^2)(q_j - q_i) + (4t - \alpha\beta)(t - \alpha\beta)}{2(t - \alpha\beta)(4t - \alpha\beta)}. \end{aligned}$$

It is worth noticing that $t > \frac{1}{4}(3\alpha\beta + \beta^2)$ for all cases. Otherwise, the demand for the magazine decreases in its own price while increases in its rival's price.

The profit function can be written as $\pi_i(q_i, q_j) = p_i(q_i, q_j) \cdot m_i(q_i, q_j) + q_i \cdot n_i(q_i, q_j)$. First order conditions with respect to q_i yield

$$R_i(q_j) = \frac{\Omega q_i + \digamma}{\Gamma}, (i = A, B),$$

where $\Gamma = \frac{256t^4 - 32t^3(\alpha^2 + 20\alpha\beta + \beta^2) + 64t^2\alpha\beta(\alpha^2 + 9\alpha\beta + \beta^2) - 2t\alpha^2\beta^2(21\alpha^2 + 110\alpha\beta + 20\beta^2) + \alpha^3\beta^3(9\alpha^2 + 30\alpha\beta + 7\beta^2)}{(2t - \alpha\beta)(4t - 3\alpha\beta - \beta^2)}$, $\Omega = 16t^2 - 4t\alpha(\alpha + 15\beta) + \alpha^2\beta(3\alpha + 5\beta)$ and $\digamma = \frac{[16t^2 - 4t\alpha(\alpha + 15\beta) + \alpha^2\beta(3\alpha + 5\beta)](4t - \alpha\beta)(t - \alpha\beta)}{4t - 3\alpha\beta - \beta^2}$.

The second order condition requires that

$$-\frac{\Gamma(2t - \alpha\beta)(4t - 3\alpha\beta - \beta^2)}{(4t - 3\alpha\beta)^2(t - \alpha\beta)(4t - \alpha\beta)^2} < 0.$$

Since $t - \alpha\beta > 0$ and $4t - 3\alpha\beta - \beta^2 > 0$, we have that $\Gamma > 0$. Magazine prices are strategic substitutes when $\Omega < 0$, that is $t \in [\frac{1}{8}(\alpha^2 + 5\alpha\beta - \alpha\sqrt{\alpha^2 - 2\alpha\beta + 5\beta^2}), \frac{1}{8}(\alpha^2 + 5\alpha\beta + \alpha\sqrt{\alpha^2 - 2\alpha\beta + 5\beta^2})]$. With another three conditions that $t > 0$, $t > \alpha\beta$ and $4t - 3\alpha\beta - \beta^2 > 0$, we derive that prices are strategic substitutes as $t \in [\max\{0, \alpha\beta\}, \frac{1}{8}(\alpha^2 + 5\alpha\beta + \alpha\sqrt{\alpha^2 - 2\alpha\beta + 5\beta^2})]$. ■

Proof of Proposition 7. The profit maximiation program yields that $\frac{\partial \pi_A}{\partial q_A} = \frac{\partial p_A}{\partial q_A} m_A + p_A \frac{\partial m_A}{\partial q_A} + q_A \frac{\partial n_A}{\partial q_A} + n_A = 0$. The impact of the rival's price on A's profit is that $\frac{\partial \pi_A}{\partial q_B} = \frac{\partial p_A}{\partial q_B} m_A + p_A \frac{\partial m_A}{\partial q_B} + q_A \frac{\partial n_A}{\partial q_B}$. Since $\frac{\partial n_A}{\partial q_B} = -\frac{\partial n_A}{\partial q_A}$, we have that

$$\begin{aligned} \frac{\partial \pi_A}{\partial q_B} &= \left(\frac{\partial p_A}{\partial q_A} + \frac{\partial p_A}{\partial q_B}\right) m_A + p_A \left(\frac{\partial m_A}{\partial q_A} + \frac{\partial m_A}{\partial q_B}\right) + n_A \\ &= -\frac{\beta}{4t - 3\alpha\beta}(m_A - p_A) + n_A \\ &= (2t - \alpha\beta)(4t - 3\alpha\beta - \beta^2)\left[\frac{1}{16t^2 - 20t\alpha\beta + \alpha^2\beta(6\alpha + \beta)} + \frac{s}{\Phi}\right]. \end{aligned}$$

In equilibrium, $n_A^T = \frac{1}{2} + \frac{16t^2 - 20t\alpha\beta + \alpha^2\beta(6\alpha+\beta)}{\Phi} s \leq 1$. Therefore,

$$\frac{\partial \pi_A}{\partial q_B} > 0.$$

We have already got that

$$sign(\frac{\partial q_B^T}{\partial s}) = sign(-\Omega).$$

Straightforward, $\frac{\partial \pi_A}{\partial q_B} \cdot \frac{\partial q_B^T}{\partial s} > 0$ when magazine prices are strategic substitutes. $\blacksquare$

References

[1] Amelio, A. and B.Jullien (2007) "Tying and Freebie in Two-sided Markets", unpublished manuscript, June 2007.

[2] Armstrong, M. (2006) "Competition in Two-sided Markets", *Rand Journal of Economics*, Vol. 37(3), 2006.

[3] Armstrong, M. and J. Wright (2005) "Two-Sided Markets, Competitive Bottlenecks and Exclusive Contracts", *Economic Theory*, forthcoming.

[4] Bulow, J. I., J. D. Geanakoplos, and P. D. Klemperer (1985) "Multimarket Oligopoly: Strategic Substitutes and Complements", Journal of Political Economy, 93(3), 488-511.

[5] Caillaud, B. and B. Jullien (2003) "Chicken & Egg: Competition Among Intermediation Service Providers", *Rand Journal of Economics* 34 (2),309-328.

[6] J. P. Choi (2007) "Tying in Two-Sided Markets with Multi-Homing", unpublished manuscript, May 2007.

[7] Evans, D. S. (2003) "The Antitrust Economics of Multi-Sided Platform Markets", *Yale Journal on Regulation*, 20(2), 325-82.

[8] Fudenberg, D. and J. Tirole (1984) "The Fat-Cat Effect, the Puppy-Dog Ploy, and the Lean and Hungry Look", *American Economic Review*, 74(2), 361-366.

[9] Farhi, E. and A. Hagiu (2007) "Strategic Interactions in Two-sided Market Oligopolies", unpublished manuscript, August 2007.

[10] Rochet, J-C. and J. Tirole (2003a) "Platform Competition in Two-Sided Markets", *Journal of the European Economic Association*, 1(4), 990-1029.

[11] Rochet, J-R. and J. Tirole (2003b) "Tying in Two-Sided Markets and the Impact of the Honor All Cards Rule", unpublished manuscript, September 2003.

[12] Rochet, J-C. and J. Tirole (2005) "Two-Sided Markets: A Progress Report", *Rand Journal of Economics.*

[13] Stennek, J. (2006) "Exclusive Quality: Why Exclusive Distribution may Benefit the TV Viewers?"

[14] Tirole, J. (1988) "The Theory of Industrial Organization", MIT Press.

[15] Whinston, M. D. (1990) "Tying, Foreclosure, and Exclusion", *American Economic Review*, September 1990, 80, 837-859.