Strategic R&D Investment, Competitive Toughness and Growth

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Abstract

We show, *within a single industry*, the possibility that R&D-investment be non-monotonically related to competitive toughness: increasing when competition is soft and decreasing when competition is tough. This possibility results from a Schumpeterian *markup squeezing effect* discouraging innovation, and a *concentration effect* spurring innovators. It is obtained in a sectoral model where the number of innovators is random and where non-successful investors may remain productive. The result is extended to a multi-sectoral stochastic endogenous growth model with overlapping generations of consumers and firms, the number of which is endogenously determined in the capital market.

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1   Introduction

The relationship between product market competition and innovation is not simple to assess, either empirically or theoretically. It varies according to the market, the industry or the innovation characteristics. Following the Schumpeterian view (Schumpeter, 1942), monopoly rent is required to support innovative activity and tougher competition on the product market has a negative impact on innovation. This conclusion is contrary to the "Darwinian" view, for which competition is needed to force firms to innovate in order to survive, or to the view that "the incentive to invent is less under monopolistic than under competitive conditions" (Arrow, 1962).[1]

Two types of theoretical models have been developed to examine the issue, tournament and non-tournament models. In tournament models, research competition is described as a stochastic "patent-race" (Reinganum, 1985 and 1989) among a fixed number of firms and, for each firm, investment in R&D-intensity (a Poisson hazard rate) increases the chances that it innovates and takes the lead temporarily, until a new innovator, possibly capitalizing on accumulated knowledge, wins and "leapfrogs" the leader. In this kind of model, more competition, in the sense of a larger number of firms, is good for innovation since discovery is then expected earlier. In non-tournament deterministic models (Dasgupta and Stiglitz, 1980), firms compete both at the research level (in order to reduce their production costs) and in the product market, and the number of firms is endogenously determined taking into account the R&D fixed cost. More competition, in the sense of a larger number of investing firms, is now bad for innovation: it implies a lower industry R&D-investment. Therefore, considering the two types of theoretical models, the Schumpeterian view can neither be generally validated nor invalidated. Empirical evidence is not conclusive either. For example, Link and Lunn (1984) exhibit a positive effect of concentration on the returns to R&D for process innovation (supporting non-tournament models) whereas Geroski (1995), Nickell (1996), Blundell, Griffith and Van Reenen (1999) find a negative effect of concentration on innovation.[2]

Of course, one should be cautious in using concentration as a measure of competitiveness. To illustrate the point, it suffices to look at the non-tournament model of van de Klundert and Smulders (1997) and their comparison of different regimes of oligopolistic competition (*i.e.* Bertrand vs. Cournot). They exhibit a "concentration effect" by which the number of firms is reduced when competition is tougher, implying larger firm size and higher rates of innovation.[3] Hence competition is a spur for innovation because it increases concentration and, hence, the incentive to innovate.[4]

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[1] Tirole (1997) calls this the "replacement effect": the profits that are *replaced* by those resulting from innovation are larger for a monopoly.

[2] For more references and a discussion of these results see Gilbert (2006).

[3] This concentration effect of competition, resulting from the endogenous reduction of the number of firms, is a known paradox for anti-trust policy. See d'Aspremont and Motta (2000).

[4] In a similar model, studying the interdependence of market structure and growth, the same positive association between competition toughness and growth is obtained by Peretto (1999), also through industry concentration.

In empirical studies, there has been another line of research, looking for a non-monotone relationship between competition and innovation, and which can be traced back to Scherer (1965, 1967) who showed that the effect of firm size on patented inventions was diminishing for large sizes[5]. An inverted-U relationship between competition and innovation was further explored[6] by Levin *et al.* (1985) and, recently, re-examined in a panel study by Aghion *et al.* (2005), who obtain a clear inverted-U shape when plotting patents against the Lerner index. Moreover, building upon previous work[7], they provide a theoretical explanation.

Our goal here is to propose an alternative theoretical model of product market competition and innovation explaining such a non-monotone relationship. This is done in a framework combining features of tournament models and non-tournament models. As in tournament models, the expected incremental gain of innovating creates the incentive for R&D-investment by firms, and each industry can be partitioned into successful and unsuccessful firms. However, a special feature of our model is that we allow for multiple simultaneous innovators. As in non-tournament models, the concentration effect plays an essential role, not only through the variation of the number of (identical) firms, but also by taking into account the distribution of market shares between successful and unsuccessful (incremental) innovators.[8]

Firms have a two-period life, competing first at the research and then at the production levels. They are all symmetric in the first period of their lifes, having equal access to technological knowledge and capital (all industries start neck-and-neck). R&D-investment by each firm determines its probability of innovating. In the second period firms compete in the product market under some competition regime belonging to a continuum between the Cournot and Bertrand regimes. As in Arrow (1962), the key notion is the "incentive to invent" represented here by the "incremental gain of innovating", that is, the difference for an investing firm between the profit it earns when successful and the one it

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[5] Dasgupta and Stiglitz (1980) referring to the earlier empirical literature surveyed in Scherer (1970) and Kamien and Schwartz (1975), already stressed that innovative activity may become negatively correlated to concentration when an industry is too concentrated.

[6] In one study the authors find a significant inverted-U relationship between industry concentration and R&D-intensity or the innovation rate. In another study, including more variables, the statistical significance of the concentration variable (C4 index) is much lower. "These econometric studies suggest that whatever relationship exists at a general economy-wide level between industry structure and R&D is masked by differences across industries in technological opportunities, demand, and the appropriability of inventions" (Gilbert, 2005, p.37).

[7] See Aghion, Harris and Vickers (1997), and Aghion, Harris, Howitt and Vickers (2001). The latter has been extended by Encaoua and Ulph (2000), allowing for the possibility that the lagging firm leapfrogs the leader without driving it out of the market, and also obtaining a non-monotone relationship between competition and innovation. Aghion, Dewatripont and Rey (1999) introduce agency considerations with non-profit maximizing firms leading to non-Schumpeterian conclusions. A synthesis of this stream of the literature is provided by Aghion and Griffith (2005).

[8] Thompson and Waldo (1994) discuss the two kinds of innovative capitalism described by Schumpeter (1928) - "competitive capitalism", under which only the innovative firm remains active in the market, and "trustified capitalism", where loosing firms may remain active - and argue that, empirically, trustified capitalism is more important than competitive capitalism.

earns when unsuccessful. This incremental gain is introduced as a function of a continuous variable representing "competitive toughness" and allowing to compare different competition regimes.[9] As a function of competitive toughness, the incremental gain of innovating is affected by two opposite effects, a negative "markup squeezing effect", clearly Schumpeterian, and a positive concentration effect. Our basic result is to show that, under some conditions, it exhibits an inverted-U shape. Under soft competition, and assuming that innovators obtain only a small cost advantage,[10] unsuccessful firms remain active at equilibrium. Then the gap between market shares of the two groups of firms increases with competitive toughness, implying that concentration as measured by an index such as Herfindahl increases (other things equal) and that the incentive to invest in R&D, evaluated by the expected incremental gain of innovating, also increases. At some level of competitive toughness, though, unsuccessful firms are eliminated, competition becomes symmetric, thus eliminating any further gain in market shares, so that the incentive to invest eventually decreases with higher competitive toughness. On this basis we show the possibility of obtaining a non-monotone relationship between R&D-investment and competitive toughness (increasing when competition is soft and decreasing when it is tough) first in a *single sector* partial equilibrium model with a fixed number of firms, then in a *multi-sectoral* endogenous growth model where the number of firms in each sector is endogenously determined.

This will allow us to compare our explanation of a non-monotone relationship between competition and innovation with the theoretical explanation of the inverted-U pattern given by Aghion *et al.* (2005). In their theoretical model, they suppose a fixed number of firms (a duopoly) in each industry, competing both at the research and the production levels, but the set of industries can be partitioned into two types of industries: those where the two firms are at the same technological level (neck-and-neck), and those where one firm leads and the other lags.[11] In a neck-and-neck industry, R&D intensity increases with product market competition because firms invest in R&D to escape competition (the "escape competition effect"). Only in an unleveled industry, can the traditional "Schumpeterian effect" dominate (and will dominate when product market competition is sufficiently tough): as there is no incentive for the leader to invest in R&D (because of an assumed automatic catching up by the follower), only the laggard firm innovates, its chosen R&D-intensity decreasing as competition becomes tougher in the product market, dissipating the rents that

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[9] See d'Aspremont, Dos Santos Ferreira and Gérard-Varet (1991), and d'Aspremont and Dos Santos Ferreira (2006), linking competitive toughness with the conduct parameter method used in the *new industrial organization* literature (Bresnahan, 1989).

[10] This supposes a small innovation step or imperfect patent protection (large spillovers).

[11] In a preliminary version of their paper, Aghion, Bloom, Blundell, Griffith and Howitt had each duopoly producing differentiated products, and the degree of product market competition was measured by the degree of product substitutability. In the published version, each firm produces the same good and the degree of product market competition is measured by one minus the degree of collusion of the two firms in a neck-and-neck industry. This is close to the model already analysed in Encaoua and Ulph (2000), where conjectural variations are used to formalize the degree of collusion in each duopoly.

can be captured after innovation. It is then by averaging R&D-intensities across all industries, that an inverted U-relationship between the (average) innovation rate and product market competition is obtained (through a "composition effect").[12]

In section 2, we introduce a one-sector model and give the definition of the oligopolistic two-stage game under different regimes of competition. We then present the basic non-monotonicity results. In section 3, we extend the model to a continuum of sectors and analyze the consequences of the basic non-monotonicity results in this general setting where the number of firms is endogenously determined. We conclude in section 4.

2   A representative oligopolistic sector

Let us start by considering a typical industry involving a set $\mathcal{N}$ of N firms. Suppose first that each firm j has already chosen its level of investment in R&D, and that uncertainty on innovation is resolved, resulting in a partition of the industry into successful and unsuccessful firms. Each firm j has to choose a price-output pair (p_j, y_j). Output y_j can be produced at unit cost c_j. This unit cost takes three values: the lowest one for successful firms, an intermediate value for non-successful firms benefitting from innovators' spillovers, and the highest value when no firms are successful. The demand D for the good is a function of market price P, with a finite negative continuous derivative over all the domain where it is positive. A particular specification of the demand function will be introduced below. We let Y denote the total output $\sum_k y_k$, and $Y_{-j} = \sum_{k \neq j} y_k$ the total output produced by firms other than j.

2.1   The oligopolistic equilibrium

Since our goal is to study the relationship between the degree of competition and R&D-investment, we want to compare different competition regimes, including Bertrand and Cournot, but also other intermediate regimes. This is common practice in the *New Empirical Industrial Organization* (NEIO) literature where "behavioral equations" are used to estimate firm behavior, in which the degree of competitiveness of a firm is represented by a conduct parameter.[13] From a theoretical point of view, there are various ways to derive these behavioral equations.[14] We shall not here review all these various theories but shall rely on the canonical, and Cournot-like, representation of oligopolistic competition introduced in d'Aspremont and Dos Santos Ferreira (2006), and readily usable in

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[12] Mukoyama (2003) also obtains an inverted-U relationship between competition and growth in a tournament model which is close to Aghion *et al.* (2005). This model introduces the possibility that a successful innovator be imitated by (at most) one other firm (following a Poisson process). When imitation has succeeded the two firms engage in an innovation race. Products are homogeneous and competition is in prices. The degree of competition is measured by the proportion of two-leader industries.

[13] For a synthesis see Bresnahan (1989).

[14] See Martin (2002)..

the present framework[15]. How this canonical representation can be derived from some more primitive theory of oligopoly will be illustrated below by a simple example. In this canonical representation each firm j is assumed to choose a price-output pair (p_j, y_j) in order to maximize its profit under two constraints representing the competitive pressure coming respectively from inside and from outside the industry. In the first constraint, firm j preserves its *market share* by matching its competitors' prices. In the second, firm j adjusts for the *market size*. At equilibrium the consumers should not be rationed. Formally, a $2N$-tuple (p^*, y^*) is an *oligopolistic equilibrium* if, for each firm j, (p_j^*, y_j^*) is solution to the program

$$\max_{(p_j, y_j) \in \mathbb{R}_+^2} \left\{ (p_j - c_j) y_j : p_j \leq \min_{k \neq j} \{p_k^*\} \text{ and } p_j \leq D^{-1} \left(y_j + Y_{-j}^* \right) \right\}, \quad (1)$$

and satisfies the no-rationing condition

$$Y^* = D(P^*), \text{ with } P^* = \min_j \{p_j^*\}. \quad (2)$$

The behavioral equations of the empirical literature coincide with the first order conditions of this program. Introducing Kuhn and Tucker multipliers $(\lambda_j, \nu_j) \in \mathbb{R}_+^2 \setminus \{0\}$ associated with the first and second constraints in (1), respectively, general first order conditions require, by the positivity of p_j^* and the nonnegativity of y_j^*, that $y_j^* - \lambda_j - \nu_j = 0$, and $p_j^* - c_j + \nu_j / D'(P^*) \leq 0$ with $\left(p_j^* - c_j + \nu_j / D'(P^*) \right) y_j^* = 0$. Defining the normalized parameter $\theta_j \equiv \lambda_j / (\lambda_j + \nu_j) \in [0, 1]$, the first order conditions for the N^* producing firms (with $p_j^* = P^*$ and $y_j^* > 0$) can then be expressed as a function of the θ_j's:

$$\frac{P^* - c_j}{P^*} = (1 - \theta_j) \frac{y_j^* / Y^*}{-\epsilon D(P^*)}, \quad (3)$$

where the left-hand side is firm j Lerner degree of monopoly and ϵ is the elasticity operator.[16] The θ_j's are determined endogenously at equilibrium and correspond to the conduct parameters used in econometric estimations. The multiplier λ_j, associated with the market share constraint, and the multiplier ν_j, associated with the market size constraint, can be interpreted as the shadow costs for firm j of relaxing the pressure coming from its competitors, respectively inside and outside the industry. Each parameter θ_j is accordingly viewed as an index of the competitiveness (or the *competitive toughness*) of firm j within the industry.

But the parameters $\{\theta_j\}$ can also be exogenously fixed and a more primitive game constructed. We give an illustrative example[17] where, to simplify the

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[15] See also d'Aspremont, Gérard-Varet and Dos Santos Ferreira (2006).

[16] For a differentiable function $f(x)$,

$$\epsilon f(x) \equiv \frac{df(x)}{dx} \frac{x}{f(x)}.$$

[17] Other examples are given in d'Aspremont and Dos Santos Ferreira (2006), respectively based on meeting-competition clauses, supply function strategies, and socially oriented managerial incentives.

argument, we assume a demand elasticity larger than or equal to 1 in absolute value. The game is in extensive form, with parameter $\theta_j \in [0,1)$ taken as the exogenous probability that firm j will adopt an *aggressive* rather than a *compromising* conduct. At the first stage, each firm j quotes (simultaneously) the maximum price p_j at which it commits to sell up to quantity y_j, to be produced in advance. Then, Nature chooses a subset $\mathcal{A}$ of aggressive firms in the set $\mathcal{N}$ of all firms according to the (independent) probabilities θ_j's. A compromising firm $j \in \mathcal{N} \backslash \mathcal{A}$ supplies at quoted price p_j a quantity x_j which is less than or equal to (in fact equality will prevail since production is in advance) the minimum of the produced quantity y_j and the residual demand, namely $\max \left[D(p_j) - \sum_{k \in \mathcal{N} \backslash \{j\}} y_k, 0 \right]$. By contrast, an aggressive firm j undercuts its rivals by an arbitrarily low exogenous amount $\varepsilon > 0$, supplying at price $\psi_j(p) \equiv \min \left[\min_{k \neq j} \{p_k\} - \varepsilon, p_j \right]$ a quantity x_j less than or equal to (in fact equal to) the minimum of what it has produced and what it can confidently expect to sell, namely the residual demand $\max \left[D(\psi_j(p)) - \sum_{k \in \mathcal{A} \backslash \{j\}} y_k, 0 \right]$ with respect to the set $\mathcal{A} \backslash \{j\}$ of aggressive rivals.[18] We look for subgame perfect equilibria with the additional condition $Y^* = D(P^*)$.

At the first stage, each firm j should take into account all possible partitions between agressive and compromising firms, and maximize the following expected profit (using the expectation operator **E** over the states in which it has an aggressive conduct)[19]:

$$
\begin{aligned}
\Pi_j(p, y, \theta) \;=\; & \theta_j \psi_j(p) \, \mathbf{E} \left(\min \left\{ y_j, \max \left[D(\psi_j(p)) - \sum_{k \in \mathcal{A} \backslash \{j\}} y_k, 0 \right] \right\} \right) \quad (4) \\
& + (1 - \theta_j) \, p_j \min \left\{ y_j, \max \left[D(p_j) - \sum_{k \in \mathcal{N} \backslash \{j\}} y_k, 0 \right] \right\} - c_j y_j.
\end{aligned}
$$

Since, at equilibrium, $\psi_j(p^*) \leq P^* = \min_k \{p_k^*\}$ and $Y^* = D(P^*)$, we obtain, for any $\mathcal{A}$, $D(\psi_j(p^*)) - \sum_{k \in \mathcal{A} \backslash \{j\}} y_k^* \geq \sum_{k \in \mathcal{N}} y_k^* - \sum_{k \in \mathcal{A} \backslash \{j\}} y_k^* \geq y_j^*$, so that

$$
\begin{aligned}
\Pi_j(p^*, y^*, \theta) \;=\; & \theta_j \psi_j(p^*) y_j^* \quad (5) \\
& + (1 - \theta_j) \, p_j^* \min \left\{ y_j^*, \max \left[D(p_j^*) - Y_{-j}^*, 0 \right] \right\} - c_j y_j^*.
\end{aligned}
$$

Now, there are two possible cases according to price p_j^* being larger than or equal to the minimum quoted price P^*. If price $p_j^* > P^* = \min_{k \neq j} \{p_k^*\}$, the second term in the RHS of equation (5) reduces to $(1 - \theta_j) \, p_j^* \max \left[D(p_j^*) - Y_{-j}^*, 0 \right]$ and should be (locally) maximized at p_j^* (since $\psi_j(p^*) = P^* - \varepsilon$). However, the residual demand $p_j \left[D(p_j) - Y_{-j}^* \right]$ is, under our assumption on demand elasticity, a decreasing function of p_j, and we obtain a contradiction. We conclude

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[18] We also admit that firm j always chooses to produce, in case of indifference, the maximal quantity y_j it can sell.

[19] So, when agressive, a firm j faces a state-dependent residual demand.

that $p_j^* = P^*$ for any j. This price must (locally) maximize

$$[\theta_j (P^* - \varepsilon) + (1 - \theta_j) p_j - c_j] [D(p_j) - Y_{-j}^*],\qquad(6)$$

so that we obtain at equilibrium the necessary first order condition:

$$\frac{P^* - c_j}{P^*} = (1 - \theta_j) \frac{y_j^*/Y^*}{-\epsilon D(P^*)} + \theta_j \frac{\varepsilon}{P^*}.\qquad(7)$$

As ε tends to zero, this condition eventually coincides with the corresponding condition (3) of the canonical representation of oligopolistic competition, implying the same set of (potential) equilibria. Cases where some of the θ_j's are equal to 1 are taken as limit cases.

Whether endogenously or exogenously determined, we shall treat the θ_j's as parameters indicating the relative degree of competition and use the first order conditions (3) to investigate their impact on R&D-investment. However, this analysis requires some simplifying assumptions.

2.2 The gain of innovating under varying toughness

A first simplifying assumption is to suppose a unit-elastic demand to the industry,

$$D(P) = \frac{A}{P},\qquad(8)$$

with A and P positive and denoting respectively the sectoral expenditure and the market price. A second simplification is to limit our analysis to the case in which competitive toughness is the same for all producing firms[20], that is, $\theta_j = \theta \in [0, 1]$ for any j. A third assumption is to let the unit cost of each firm depend on its type (successful or unsuccessful) and to take into account the possibility of incomplete appropriability by the innovators through a spillover coefficient σ, $0 \le \sigma < 1$. Formally, for $c(.)$ a strictly decreasing function, we let

$$c_j = c(\delta_j),\qquad(9)$$

with $\delta_j = 1$, if j is successful, $\delta_j = 0$ if no firm succeeds and $\delta_j = \sigma$, if at least one firm succeeds but not firm j. We denote by $\kappa \equiv (c(\sigma) - c(1))/c(\sigma)$ the *relative cost advantage* of the innovators. Clearly, more spillovers, or less appropriability of invention, decreases the relative cost advantage. Under these assumptions, the first order condition (3) becomes:

$$1 - \frac{c(\delta_j)}{P^*} = (1 - \theta) \frac{y_j^*}{Y^*},\qquad(10)$$

with $Y^* = A/P^*$.

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[20] Competitive toughness θ, as now defined, satisfies the axioms characterizing a measure of the intensity of competition according to Boone (2001) definition (except for the normalization axiom associating a zero θ with local monopoly).

Consider first the symmetric case where firms are all unsuccessful. The equilibrium price, as a function of the number n of successful firms, the number N of competing firms and of competitive toughness θ, is easily computed to be

$$P(0, N, \theta) = \frac{Nc(0)}{N - (1 - \theta)}. \tag{11}$$

A second case is when n firms succeed $(1 \leq n \leq N)$ and all N firms are active at equilibrium. We then obtain the equilibrium price

$$P(n, N, \theta) = \frac{nc(1) + (N - n)c(\sigma)}{N - (1 - \theta)}, \tag{12}$$

provided $P(n, N, \theta) \geq c(\sigma)$, or $\kappa \leq (1 - \theta)/n$. For $\kappa > (1 - \theta)/n$, the innovation is drastic, unsuccessful firms are eliminated, and we obtain the third case, where only successful firms are active (getting equal market shares), the price becoming:

$$P(n, n, \theta) = \frac{nc(1)}{n - (1 - \theta)}. \tag{13}$$

Observe that there is a borderline case where the competitive toughness is just sufficient to eliminate unsuccessful firms, i.e.

$$\theta = \theta^L(n) \equiv \max\{1 - n\kappa, 0\}. \tag{14}$$

We call this borderline case the *limit-pricing regime*. The corresponding θ varies with n. It can be extended to the cases in which there are no successful firms and in which all firms succeed, by taking the limit values of $\theta^L(n)$, denoted respectively $\theta^L(0)$ (equal to 1 and corresponding to the Bertrand outcome) and $\theta^L(N)$.

For the following analysis, it is convenient to introduce a specific notation for market shares. For $0 < n \leq N$, we let $m(1, n, N, \theta)$ and $m(\sigma, n, N, \theta)$ denote respectively the market share of a successful and an unsuccessful firm. From equation (10) and the price equations, it is easy to verify that the equilibrium market share is:

$$\frac{y_j^*}{Y^*} = m(\delta_j, n, N, \theta) = \frac{1}{1 - \theta}\left(1 - \frac{N - (1 - \theta)}{nc(1) + (N - n)c(\sigma)}c(\delta_j)\right) \tag{15}$$

or, using the notation κ and taking into account the case of drastic innovations (with $\kappa > (1 - \theta)/n$),

$$m(1, n, N, \theta) = \min\left\{\frac{1}{1 - \theta}\frac{(N - n)\kappa + (1 - \theta)(1 - \kappa)}{N - n\kappa}, \frac{1}{n}\right\}, \tag{16}$$

$$m(\sigma, n, N, \theta) = \max\left\{\frac{1}{1 - \theta}\frac{1 - \theta - n\kappa}{N - n\kappa}, 0\right\}. \tag{17}$$

When $n = 0$, we let $m(\sigma, 0, N, \theta) = m(0, 0, N, \theta) = 1/N$.

Clearly $m(1, n, N, \theta) > m(\sigma, n, N, \theta)$: the market share of a successful firm is bigger than the market share of an unsuccessful one. Notice also that, for $0 < n < N$ and $\theta < \theta^L(n)$, that is, when, given the competitive toughness, innovations are non drastic, the market share $m(1, n, N, \theta)$ is increasing whereas $m(\sigma, n, N, \theta)$ is decreasing in θ, so that the gap in market shares between successful and unsuccessful firms increases with competitive toughness. In particular, the Herfindahl concentration index[21] is increasing in θ. It is through this effect on market shares, hence by enhancing concentration (as measured by the Herfindahl index) that tougher competition can stimulate R&D. Note that the concentration effect exhibited here is not measured by the reduction in the number of firms (at zero profit equilibrium under free entry) when competition becomes tougher, as it is the case in non-tournament models with all firms identical (*e.g.* van de Klundert and Smulders, 1997). We have to use a more sophisticated notion of concentration since, although we assume symmetry *ex ante*, we lose it *ex post* when there are successful and unsuccessful firms.

Using the first order condition (10), the equilibrium profit Π per unit of expenditure of firm j is then equal to

$$\Pi(\delta_j, n, N, \theta) = (1 - \theta) m(\delta_j, n, N, \theta)^2. \qquad (18)$$

We see that an increase in competitive toughness has two effects on a firm equilibrium profit, first a negative *markup squeezing effect* through $(1 - \theta)$, and second an effect through the variation in market share $m(\delta_j, n, N, \theta)$. When all active firms have the same cost, so that they all get the same market share at equilibrium, profits decrease as competition becomes tougher. This is also true as regards the profit of an unsuccessful firm ($\delta_j = \sigma$, with $n \geq 1$) since its market share is a decreasing function of competitive toughness. But, as competition becomes tougher, the increasing market share of a successful firm ($\delta_j = 1$) has a positive effect which may more than compensate the negative markup squeezing effect, provided its relative cost advantage κ is strong enough, but not so as to make the innovation drastic (see (16)).

However, as we shall see in the following, it is not so much the equilibrium profit that determines a firm incentive to innovate, but rather the incremental gain G of innovating

$$G(n, N, \theta) \equiv \Pi(1, n+1, N, \theta) - \Pi(\sigma, n, N, \theta), \qquad (19)$$

for $0 \leq n < N$ (using the equality $\Pi(\sigma, 0, N, \theta) = \Pi(0, 0, N, \theta)$). Here an increase in competitive toughness has clearly two opposite effects. There is still

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[21] The Herfindahl index is defined as

$$\mathcal{H} = \sum_{j=1}^{N} m_j^2 = \frac{1}{N} + NV,$$

with m_j the market share of firm j and the variance

$$V = \frac{1}{N} \sum_{j=1}^{N} \left(m_j - \frac{1}{N} \right)^2.$$

the negative *markup squeezing effect*, together with a positive *concentration effect* through the difference in the squares of market shares. We may thus expect to obtain a non-monotonic relationship between the incremental gain of innovating G and the competitive toughness θ. This is formalized in the following lemma showing that G either has an inverted-U shape or is decreasing for all θ.

Lemma 1 *If the relative cost advantage κ (or the number n) of successful firms is small enough, then the incremental gain of innovating G is increasing in the competitive toughness θ for sufficiently small, and decreasing for sufficiently large, values of θ. Otherwise, G is decreasing in θ in the whole interval $[0, 1]$. In any case, the function G is strictly quasi-concave in θ in the whole interval $[0, 1]$. Moreover G is decreasing in n for small enough values of κ.*

Proof. See Appendix. ∎

In the case of drastic innovations, that is, when $\theta^L(n) = 0$ due to a combination of many innovators and a large relative cost advantage, laggards are eliminated even under Cournot competition, so that the concentration effect vanishes and tougher competition can only have a negative effect on the incentive to innovate through the markup squeeze. In the case of non drastic innovations, the concentration effect is positive but weak when innovation is sufficiently appropriable (with a small spillover coefficient σ, resulting in a large relative cost advantage κ) and when the number n of innovators is large. The incentive to innovate is then stronger even if competition is soft, in conformity with Schumpeter's view. By contrast, with small values of κ and n, the incentive to innovate is first increasing and eventually decreasing with competitive toughness, so that we may obtain a non-monotone curve relating the incremental gain of innovating to competitive toughness. This non-monotonicity derives from a strong concentration effect when competition is soft, that is, from the incentive created by a high prospective increase of the innovator's market share, an effect that entirely depends upon the probabilistic nature of the model, and disappears as soon as innovation is approached as a deterministic process. The last statement of the lemma asserts that the incentive to innovate decreases with the number of innovators, a property which will play a crucial role in the following (see the proof of Proposition 1), and which relies on the possibility of multiple winners, usually excluded in tournament models. Hence, uncertainty and multiplicity of simultaneous innovations are crucial to get a non-monotone relation between competitive toughness and R&D-investment.

2.3 Strategic R&D-investment

In the preceding analysis, we have supposed that each firm had already chosen its level of investment in R&D and that uncertainty of innovation was resolved, implying that the sets of successful and of unsuccessful firms were fixed. We now introduce a first stage during which each firm j chooses a level of R&D-investment leading to discovery with some probability of success s_j. Innovation

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is assumed to be a Bernoullian random process which depends upon these invest-
ment levels. The two-stage game will depend on the expected expenditure A,
on the number of firms N and, of course, on the selected competitive toughness
θ, and will be denoted $\Gamma(A, N, \theta)$.

For simplicity, firm j investment is directly represented by the independent
probability s_j of success in the next period. We specify the investment cost
to be quadratic: $C(s_j) = \phi + (\gamma/2)\, s_j^2$ for $s_j \geq 0$ (with $\phi > 0$ and $\gamma > 0$).
The sunk cost ϕ corresponds to the investment which is required for having
access to the technology and for benefitting from technological spillovers. The
probability that a subset S of firms innovate simultaneously, while firms in
the complementary subset $\overline{S}$ do not succeed, is given by $\prod_{j \in S} s_j \prod_{j \in \overline{S}} (1 - s_j)$.
We denote $\Pr\{\nu \mid \mathbf{s}_{-j}\}$ the probability of having ν innovators among the $N-1$
competitors of firm j with investment strategies $\mathbf{s}_{-j} \equiv (s_1, ..., s_{j-1}, s_{j+1}, ..., s_N)$.

Investment s_j is decided by maximizing the profit expectation $\tilde{\Pi}(s_j, \mathbf{s}_{-j})\, A$,
for given values of A, N and θ, with

$$
\tilde{\Pi}(s_j, \mathbf{s}_{-j}) = s_j \sum_{\nu=0}^{N-1} \Pr\{\nu \mid \mathbf{s}_{-j}\} \Pi(1, \nu+1, N, \theta)
$$

$$
+ (1 - s_j) \sum_{\nu=0}^{N-1} \Pr\{\nu \mid \mathbf{s}_{-j}\} \Pi(\sigma, \nu, N, \theta) - \frac{C(s_j)}{A}. \tag{20}
$$

This expectation is strictly concave in the strategy variable s_j, by the specifica-
tion of the cost function. We thus obtain for each j the necessary and sufficient
first order condition for an interior maximum (at $s_j \in (0,1)$)

$$
\begin{aligned}
\frac{C'(s_j)}{A} &= \sum_{\nu=0}^{N-1} \Pr\{\nu \mid \mathbf{s}_{-j}\} \left[\Pi(1, \nu+1, N, \theta) - \Pi(\sigma, \nu, N, \theta)\right] \\
&= \sum_{\nu=0}^{N-1} \Pr\{\nu \mid \mathbf{s}_{-j}\}\, G(\nu, N, \theta),
\end{aligned}
$$

the equality of the marginal R&D-investment cost and of the expected value of
the incremental gain of innovation (both by unit of expenditure).

At a symmetric equilibrium (with $s_j = s$, for any j), this first order condition
becomes:

$$
\frac{\gamma s}{A} = \sum_{\nu=0}^{N-1} \Pr\{\nu \mid (s, ..., s)\}\, G(\nu, N, \theta) \equiv \overline{G}(s, N, \theta), \tag{21}
$$

where

$$
\Pr\{\nu \mid (s, ..., s)\} = \frac{(N-1)!}{(N-1-\nu)!\nu!} s^\nu (1-s)^{N-1-\nu}. \tag{22}
$$

By continuity of $\overline{G}$ as a function of s, and since $\overline{G}(0, N, \theta) = G(0, N, \theta) > 0$,
either there is a value $s \in (0,1)$ satisfying equation (21), or $\overline{G}(s, N, \theta) > \gamma/A$
for any $s \in (0,1)$ and the corner solution $s = 1$ applies.

Using Lemma 1 and the probabilities given by (22), we may easily derive
the following conclusions:

Lemma 2 *If the relative cost advantage κ of successful firms (or the probability of success s) is small enough, then the expected incremental gain of innovating $\overline{G}(s, N, \theta)$ is increasing with the competitive toughness θ for sufficiently small, and decreasing for sufficiently large, values of θ. Otherwise, $\overline{G}$ is decreasing in θ in the whole interval $[0, 1]$.*

Proof. Given N, $\overline{G}(s, N, \theta)$, as a function of θ, is an expectation (determined by s) computed from the set of functions $G(\nu, N, \theta)$ for $\nu = 0, 1, ..., N-1$. By Lemma 1, whenever κ is small enough, every such $G(\nu, N, \theta)$ is increasing in θ for any θ close to 0. Also, for s small enough, we see by (22) that most of the weight is put on small values of v entailing (again by Lemma 1) that $G(\nu, N, \theta)$ is increasing in θ for any θ close to 0. In both cases (small κ or small s), $\overline{G}(s, N, \theta)$ is increasing in θ for any θ close to 0. Since, for any ν, $G(\nu, N, \theta)$ is decreasing in θ for any θ close to 1 (and for all θ in $[0, 1]$ when neither κ nor s are too low), $\overline{G}(s, N, \theta)$ inherits the same property. ∎

Except for quasi-concavity, the properties exhibited in Lemma 1 for the function G, are preserved in expected terms for the function $\overline{G}$, with a condition imposing a small probability of success replacing the condition of a small number of successful firms. An implication is that, when the relative cost advantage of innovation is significant, the possibility of deviating from a strict Schumpeterian view arises from the probabilistic nature of the model.

The following figure illustrates Lemma 2 by showing how the expected incremental gain varies with competitive toughness for three values of the probability of success s ($s = 0.25, 0.5, 0.75$).[22] For the lowest value of s, we obtain a hump-shaped curve dominating the other two. For the largest value the curve is decreasing overall.
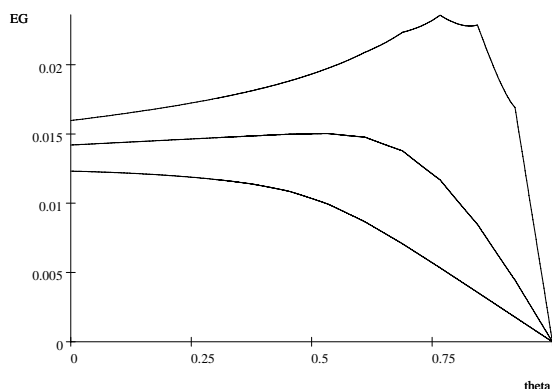

Figure 1

The next proposition describes properties of the (symmetric) equilibrium where all firms choose the (same) level of R&D-investment, a probability of success s, for various values of competitve toughness.

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[22] The figure is based on the parameter values $N = 9$ and $\kappa = 0.078$.

Proposition 1 *If the relative cost advantage κ of successful firms is small enough, the symmetric equilibrium level s of R&D-investment is uniquely determined for every value of θ. If, in addition, the slope γ/A of the marginal investment cost per unit of aggregate expenditure is large enough (so as to exclude a corner solution), then s is increasing with competitive toughness θ for sufficiently small, and decreasing for sufficiently large, values of θ. Moreover, s is decreasing in γ/A.*

Proof. By Lemma 1, the incremental gain G is decreasing in n for small enough values of κ. An implication of this property of G is that the expected incremental gain $\overline{G}$ is decreasing in s. Indeed, by (22), the elasticity of the weight $\Pr\{\nu \mid (s, ..., s)\}$ with respect to s is equal to $(\nu - (N-1)s)/(1-s)$, which has the sign of the excess of ν over its mean. Thus, an increase in s displaces the mass towards the terms corresponding to a larger number of successful firms, those for which the incremental gain is lower. It results that the first order condition for equilibrium investment (21) uniquely determines the symmetric equilibrium value of s at the intersection of the decreasing curve on the right hand side with the increasing line on the left hand side, provided this line has a large enough slope γ/A (otherwise, we obtain the corner solution $s = 1$). Then, from Lemma 2, for a sufficiently low competitive toughness, the expected incremental gain is increasing, implying by (21) an increasing equilibrium level s of R&D-investment. The reverse holds for sufficiently soft competition. Finally, notice that the equilibrium value s (a solution to (21)) is smaller for a higher slope γ/A. ∎

The proof of Proposition 1 is illustrated in Figure 2, where the increasing line corresponds to the LHS of (21), and the decreasing curves to its RHS, for values of the competitive toughness $\theta = 0$ and $\theta = 0.12$ (the upper thick and thin curves, respectively), and $\theta = 0.8$ and $\theta = 0.9$ (the lower thick and thin curves, respectively).[23]
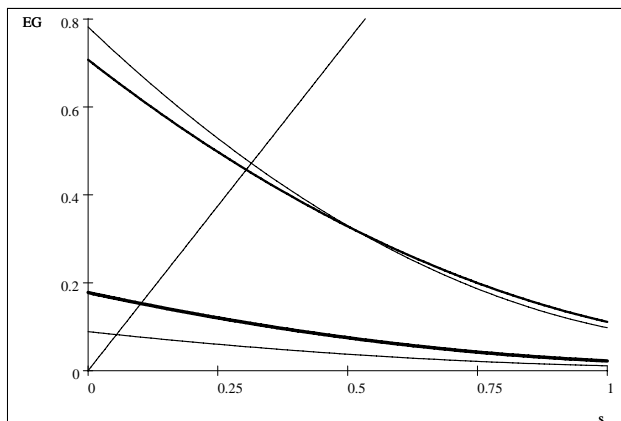

Figure 2

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[23] The figure is based on the parameter values $N = 3$, $\kappa = 0.1$ and $\gamma/A = 1.5$.

In the situation depicted in Figure 2, the equilibrium level s of R&D-investment increases (resp. decreases) as one switches from the thick to the thin curve, that is, as competition becomes tougher, starting from soft (resp. tough) competition. The reverse result would be attained, in the case of soft competition, for lower values of the slope γ/A of the increasing line (since the two decreasing curves intersect).

Notice that Proposition 1 uses the main properties of the incremental gain function that were exhibited in Lemma 1, in particular the fact that it decreases with the number of innovators, which has the consequence that the expected incremental gain is decreasing in s. This means that assuming the number of innovators to be a random variable, following a Bernoullian process, is essential to establish the proposition.

3   Competition and innovation in endogenous growth: a non-monotone relationship

The previous proposition shows the possibility of obtaining a non-monotone relationship between R&D-investment and competitive toughness (increasing at low values and decreasing at high values), by using a *single sector* partial equilibrium model. As mentioned in the introduction, an inverted-U relationship has been theoretically derived by Aghion *et al.* (2005) in a *multi-sectoral* endogenous growth model, through a "composition effect of competition on the steady-state distribution of technology gaps across sectors". When competition increases from its minimal to its maximal level, the aggregate innovation rate first increases (the "escape competition effect" dominates because of a larger percentage of neck-and-neck industries) and then decreases (the "Schumpeterian effect" dominates because of a larger percentage of industries exhibiting a technology gap).

We shall show that the one-sector model constructed above and the associated two-stage game can serve as a building block in an endogenous growth general equilibrium model with a continuum of uniformly distributed oligopolistic industries and that the non-monotone pattern can also be predicted for cross-section investigations. In our model, though, we do not assume that every industry has a fixed number of firms (two in Aghion *et al.*, 2005). It is composed of a finite but endogenously determined number of firms, each having a two-period life, competing first at the research and then at the production levels. The number of successful firms is a random variable, the realisation of which may differ across industries.

3.1   A model with overlapping generations of firms and consumers

We use an overlapping generations model with a continuum of produced goods and an infinite number of periods ($t = 0, 1, ...$). Both firms and consumers live

for two periods, corresponding to R&D-investment and production stages for firms, and young and old ages for consumers.

On the firms side, there is a continuum of identical oligopolistic industries of mass 1. At date $t + 1$, there is in each industry i a number N_{it} ($N_{it} \geq 2$) of firms created at date t, which produce good i for immediate consumption, on a one-to-one basis with respect to effective labor, supplied by young consumers:

$$y_{ijt+1} = l_{ijt+1} H_{it} \eta^{\delta_{ijt}},$$

where y_{ijt+1} and l_{ijt+1} are the output and labor input of firm j, respectively, H_{it} is the inherited stock of knowledge (available to the whole industry at the beginning of period t), and $\eta > 1$ is the *innovation step* if the firm has succeeded to innovate (with probability s_{ijt} and cost $C(s_{ijt})$) at the end of period t. As indicated in the preceding section, δ_{ijt} is equal to 1 for an innovator, to $\sigma \in [0, 1)$ for an unsuccessful firm benefitting from spillovers from successful competitors, and to 0 if there were no innovators in period t. Accordingly, and taking labor as the numeraire, the unit production cost is $c_{it}(\delta_{ijt}) = 1/H_{it}\eta^{\delta_{ijt}}$, so that the innovators' relative cost advantage is a constant $\kappa = 1 - \eta^{\sigma-1}$. We also assume that public knowledge accumulates in proportion to the percentage of successful firms:

$$\frac{H_{it+1} - H_{it}}{H_{it}} = (\eta - 1)\frac{n_{it}}{N_{it}}. \tag{23}$$

On the consumers side, there is a continuum of identical consumers of constant unit mass at each generation. One unit of labor is inelastically supplied at wage 1 by each young consumer, who has to choose present consumption $x_t \in \mathbb{R}_+^{[0,1]}$ and saving $z_t \in \mathbb{R}_+$, under the budget constraint[24] $\langle P_t, x_t \rangle + z_t = 1$, where $P_t \in \mathbb{R}_+^{[0,1]}$ is the vector of market prices. Anticipated future consumption $\widetilde{x}_{t+1}$ is a random variable induced by $\widetilde{r}_{t+1}$ and $\widetilde{P}_{t+1}$ according to the budget constraint $\left\langle \widetilde{P}_{t+1}, \widetilde{x}_{t+1} \right\rangle = \widetilde{r}_{t+1} z_t$, where $\widetilde{P}_{t+1}$ is the vector of anticipated market prices and $\widetilde{r}_{t+1}$ is the expected return factor on capital. Saving is supposed to be invested in funds that allow to cancel out idiosyncratic risks, but not aggregate risk, so that $\widetilde{r}_{t+1}$ is a random variable depending upon the success of the innovative efforts by all investing firms. For simplicity, we assume symmetric log-linear sub-utility functions:

$$U(x_t, \widetilde{x}_{t+1}) = \alpha \int_0^1 \ln x_{it} di + (1 - \alpha) \int_0^1 \ln \widetilde{x}_{it+1} di, \text{ with } \alpha \in]0, 1[.$$

so that $x_{it} = (1 - z_t)/P_{it}$ and $\widetilde{x}_{it+1} = \widetilde{r}_{t+1} z_t / \widetilde{P}_{it+1}$. Maximizing expected utility reduces to maximizing $\alpha \ln(1 - z_t) + (1 - \alpha) \ln z_t$, leading to the solution $z_t = 1 - \alpha$.

Given r_t (the actual return factor) and P_t, old consumers at period t optimally choose consumptions $x'_{it} = r_t z_{t-1}/P_{it} = (1 - \alpha) r_t/P_{it}$. Thus, adding

---

[24] Given P and x belonging to $\mathbb{R}^{[0,1]}$, we let $\langle P, x \rangle$ denote the inner product $\int_0^1 P_i x_i di$.

consumptions by youngs and olds, we obtain the aggregate demand A_t/P_{it} for good i, with aggregate expenditure

$$A_t = \alpha + (1 - \alpha)r_t. \tag{24}$$

Observe that, as we have assumed a continuum of sectors, A_t is unaffected by sectoral idiosyncratic variations.

3.2 The intertemporal stochastic equilibrium

The N_{it} firms in industry i, created at date t, can be seen as involved in a two-stage game $\Gamma(A_{t+1}, N_{it}, \theta_{it+1})$ of the kind analysed in section 2. In the first stage of this game, corresponding to the investment period t, each firm j, producing good i, chooses strategically a probability s_{ijt} of success and accordingly invests $C\left(s_{ijt}\right)$ in R&D. Uncertainty on innovation is resolved at the end of period t, resulting for each industry i in a number n_{it} of successful firms. In the second stage, corresponding to the production period $t + 1$, each firm j chooses a price-output pair (p_{ijt+1}, y_{ijt+1}). Our concept of equilibrium for the whole economy is based on the sequence $\left(\mathbf{s}_{it}, \left(p_{it+1}^*, y_{it+1}^*\right)\right)_{it}$ of solutions to the sequence $(\Gamma(A_{t+1}, N_{it}, \theta_{it+1}))_{it}$ of these two-stage games, for all industries. We assume such solutions to be symmetric within each sector relative to investing firms ($\mathbf{s}_{it} = (s_{it}, ..., s_{it})$) and within each category of producing firms (successful and unsuccessful).

This sequence of solutions is determined by the sequence of vectors $(\theta_{it+1})_{it}$ of competitive toughness which can be treated as exogenously given (except in the limit pricing regime, where $\theta_{it+1} = \theta^L(n_{it})$). In order for this sequence of solutions to deliver an *intertemporal stochastic equilibrium*, the sequence of values of the variables $(n_{it}, N_{it}, A_{t+1})$ must satisfy three sequences of conditions. The first sequence of conditions corresponds to the first order conditions for *equilibrium investment* (see (21)):

$$\frac{\gamma}{A_{t+1}} s_{it} = \overline{G}\left(s_{it}, N_{it}, \theta_{it+1}\right). \tag{25}$$

The second sequence of conditions corresponds to *capital market clearing*

$$\int_0^1 N_{it} C\left(s_{it}\right) di = 1 - \alpha, \tag{26}$$

expressing the equality of aggregate R&D-investment and aggregate saving. Finally, the third sequence of conditions corresponds to *labor market clearing*. Total labor supply is 1 but, by the capital market clearing conditions, a proportion $1 - \alpha$ of labor is employed by investing firms, leaving α to producing firms, so that we get

$$A_{t+1} \int_0^1 L\left(n_{it}, N_{it}, \theta_{it+1}\right) di = \alpha \tag{27}$$

where $L\left(n_{it}, N_{it}, \theta_{it+1}\right)$ is labor demand per unit of expenditure in industry i. As the wage is normalized to 1, labor demand must be equal at equilibrium to

expenditure minus the total profits of successful and unsuccessful firms, so that, by (18), we obtain the following expression:[25]

$$
\begin{aligned}
L\left(n_{it}, N_{it}, \theta_{it+1}\right) = {} & 1 - n_{it}\left(1 - \theta_{it+1}\right) m\left(1, n_{it}, N_{it}, \theta_{it+1}\right)^2 \\
& - \left(N_{it} - n_{it}\right)\left(1 - \theta_{it+1}\right) m\left(\sigma, n_{it}, N_{it}, \theta_{it+1}\right)^2. \quad (28)
\end{aligned}
$$

In order to compare the implications of our model with the cross-sectional observations of Aghion *et al.* (2005), we refer to the first order conditions for equilibrium investment (25) in two industries of the same size, where the coefficient (γ/A_{t+1}) on the left hand side of (25) and the relative cost advantage κ are the same. As a direct corollary of Proposition 1, we may derive the following cross-section result.

Proposition 2 *Consider an intertemporal stochastic equilibrium and any period t. Suppose that two industries i and i' have the same size ($N_{i't} = N_{it}$), the same sufficiently low relative cost advantage κ and the same sufficiently steep marginal investment cost per unit of expenditure (high γ/A_{t+1}). Also assume that both industries have a sufficiently low (resp. high) competitive toughness, but that i' is more competitive than i: $\theta_{i'} > \theta i$. Then the R&D-investment of the more competitive sector is larger (resp. smaller): $s_{i'} > s_i$ (resp. $s_{i'} < s_i$).*

In other words, a statistical cross-section of otherwise identical industries (with low relative cost advantage and steep marginal investment cost) should reveal that if competition is soft (resp. tough) for two of them, the more competitive one invests more (resp. less).

3.3 The implications of an endogenous number of firms

Aghion *et al.* (2005) limit their analysis to an economy where every product market is a duopoly. By contrast, van de Klundert and Smulders (1997) introduce a non-tournament model, where the number of firms is endogenously determined under free entry by the zero profit condition. This model allows them to show that the (differentiated) Bertrand equilibrium always implies a higher rate of innovation than the Cournot equilibrium. This is so because tougher competition, meaning lower markups and prices, enlarges the market for high-tech goods and weakens the relative weight of R&D costs, increasing the attractiveness of R&D-investment. But, at the same time, tougher competition also reduces the equilibrium number of firms, implying larger firm size and more means devoted to R&D activity. Although our model is a tournament model, we shall see that a comparison with the approach of van de Klundert and Smulders (1997) is straightforward.

In order to achieve this comparison, we assume identical regimes of competition across time and sectors ($\theta_{it} = \theta$ for any i and t), the symmetry of the

---

[25] Observe that labor demand can be expressed as

$$
L\left(n_{it}, N_{it}, \theta_{it+1}\right) = 1 - \left(1 - \theta_{it+1}\right) \mathcal{H}\left(n_{it}, N_{it}, \theta_{it+1}\right),
$$

where $\mathcal{H}\left(n_{it}, N_{it}, \theta_{it+1}\right)$ is the Herfindahl concentration index for industry i at date $t+1$.

model leading to stochastic equilibria that are symmetric and quasi-stationary, *i.e.* with random variables $(\tilde{n}_{it})_{it}$ following the same binomial law of constant parameters (N, s).[26] The *capital market clearing condition* then simplifies to

$$NC\left(s\right) = N\left(\phi + \frac{\gamma}{2}s^2\right) = 1 - \alpha. \tag{29}$$

Also, the labor market clearing condition can be re-written (using the weak law of large numbers) as

$$A\sum_{\nu=0}^{N}\frac{N!}{(N-\nu)!\nu!}s^\nu\left(1-s\right)^{N-\nu}L\left(\nu, N, \theta\right) \equiv A\overline{L}\left(s, N, \theta\right) = \alpha. \tag{30}$$

Combining this condition with the first order condition (25) for equilibrium investment, we obtain the *equilibrium investment condition*

$$\frac{\gamma}{\alpha}\overline{L}\left(s, N, \theta\right)s = \overline{G}\left(s, N, \theta\right). \tag{31}$$

The equilibrium level of the probability s and the equilibrium (average) number N of firms can be determined by solving equations (29) and (31),[27] for given competitive toughness θ (or by taking $\theta^L\left(\nu\right)$ as the third argument of $L\left(\nu, N, \theta\right)$ in (30) if we consider the limit pricing regime). Once s and N are determined, all other variables can be readily computed, in particular the expected growth rate $(\eta - 1)s$.

In order to further facilitate the comparison between the two approaches, we take an example, solving our model numerically for specific values of the parameters: $\eta = 1.1$, and $\sigma = 0.15$ for the innovation step and the spillover coefficient, respectively (leading to the relative cost advantage $\kappa = 1 - \eta^{\sigma-1} = 0.078$), $\alpha = 0.75$ for the propensity to consume, $\gamma = 0.03$ for the variable investment cost and the two values $\phi_1 = 0.025$ and $\phi_2 = 0.055$ for the fixed cost. Figure 3 gives a geometrical representation of this example.

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[26] The number of investing firms in each industry depends upon the way savings are allocated to firms in the capital market. Here we suppose that this allocation results in a common number N of firms. However, to be precise, the value of N resulting from the equilibrium conditions is not necessarily an integer, so that it should be seen as a weighted average of the (integer) numbers of firms in the different industries, for instance of the two integers that are closest to N.

[27] The ZP (zero profit) curve in van de Klundert and Smulders (1997) plays in fact a role analogous to the curve expressing the capital market clearing condition (29) in our model. Also, their CME (capital market equilibrium) curve – resulting in particular from conditions for profit maximizing relative to (non-strategic) investment decisions – plays in their model a role equivalent to the curve expressing the equilibrium investment condition (31) in our model.
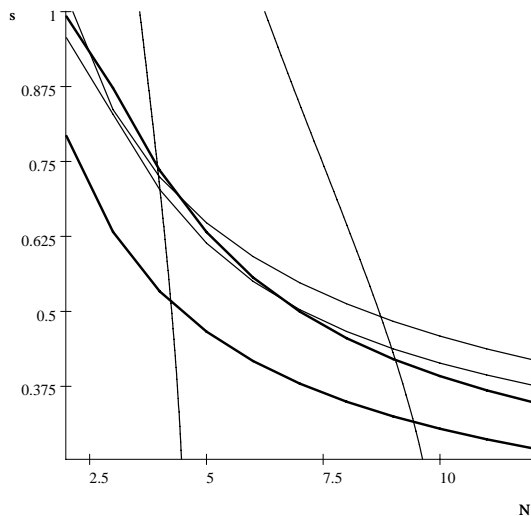
Figure 3

The equilibrium levels of s and N are given by the intersection of one of the two steep *capital market clearing* curves (the one to the right for the low value ϕ_1 and the one to the left for the high value ϕ_2) with one of the four flatter *equilibrium investment* curves (for different values of competitive toughness θ). The four equilibrium investment curves correspond to $\theta = 0$ (Cournot regime, upper thick curve), $\theta = .5$ (lower thin curve), $\theta = .9$ (close to Bertrand regime, lower thick curve), and limit pricing (upper thin curve). The relationship between R&D effort, as represented by the probability of success s, and competitive toughness is monotone decreasing in the case of high fixed cost ϕ_2 (left steep curve). The equilibrium value of s decreases indeed with competitive toughness from $s \simeq 0.75$ (for $\theta = 0$) through $s \simeq 0.69$ (for $\theta = 0.5$) to $s \simeq 0.52$ (for $\theta = 0.9$) – limit pricing giving an intermediate value $s \simeq 0.73$ – with an increasing average number of investing firms close to 4. The sense of this relationship, resulting from a concentration effect too small to dominate the markup squeezing effect, conforms with the Schumpeterian prediction. It contradicts van de Klundert and Smulders'conclusions.[28]

Besides, according to Proposition 1, the relationship between R&D effort and competitive toughness is non-monotone in our model for parameter values entailing lower equilibrium probabilities of success. Indeed, in the case of a low fixed cost ϕ_1 (right steep curve), leading to smaller equilibrium probabilities s (and to higher numbers of investing firms, fluctuating between 8 and 10), s first increases with competitive toughness from $s \simeq 0.42$ (for $\theta = 0$) to $s \simeq 0.44$ (for $\theta = 0.5$) and then falls back to $s \simeq 0.32$ (for $\theta = 0.9$) – limit pricing entailing the highest value $s \simeq 0.49$. This non-monotonicity is possible because the

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[28] In their model, they have two sectors, one producing high-tech differentiated goods, where innovation takes place, and another perfectly competitve sector. The positive concentration effect of tougher competition is reinforced by an increase in the high-tech market size (since the relative price of high-tech goods goes down). This feature is absent in our model.

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equilibrium investment curves for $\theta = 0$ and $\theta = 0.5$ intersect in this example. As already emphasized, it is in accordance with the inverted-U pattern empirically found by Aghion *et al.* (2005).

A final observation is in order. When the concentration effect of an increase in competitive toughness θ dominates the markup squeezing effect, so that the equilibrium investment curve is locally shifting upwards, the average number N of investing firms must decrease, because the capital market clearing curve is decreasing. This means that the concentration effect prevalent at the sectoral partial equilibrium level is in this case reinforced at the general equilibrium level.

4   Conclusion

Incentives to innovate depend upon multiple and conflicting effects, and it is only natural that there is no clear-cut answer to the question of determining if tough competition tends to spur or deter potential R&D investors. By combining features of tournament and non-tournament models, more specifically by admitting the possibility for investing firms either to innovate along with some of their rivals, or to fail in their R&D effort and yet to remain productive, we have obtained a relationship between innovation and product market competition which may well be non-monotone for an individual industry. This result does not rely on a composition effect, as it is the case for the inverted-U pattern of Aghion *et al.* (2005). Non-monotonicity follows straightforwardly from the interplay of two conflicting effects: the negative Schumpeterian effect of tougher competition through markup squeeze and the positive concentration effect expanding innovators' market shares. For the latter to dominate the former, one must assume non drastic innovations (a small relative cost advantage of the innovators), so that unsuccessful firms keep a positive market share, and conditions for a low equilibrium probability of R&D success, so that the incremental gain of those which do succeed is high enough to encourage R&D effort (and the more so the tougher the competition). It should be noticed that the concentration effect at work in our model is not primarily related to the reduction of the number of firms (as in van de Klundert and Smulders, 1997, and other symmetric non-tournament models) but rather to the increase in innovators' market shares. But higher competitive toughness is favorable to innovation only when competition is initially soft, otherwise one obtains the traditional Schumpeterian deterring effect of competition on innovative activity.

A   Appendix: Proof of Lemma 1

We start by studying the function $G(n, N, \theta)$, the expression of which differs in three intervals: $\left[0, \theta^L(n+1)\right]$, $\left[\theta^L(n+1), \theta^L(n)\right]$ and $\left[\theta^L(n), 1\right]$, with $\theta^L(n) = \max\{1 - n\kappa, 0\}$. Using (16), (17), (18) and (19), we obtain for $\theta \in$

$\left[0, \theta^L (n+1)\right]$:

$$\begin{aligned}
G(n, N, \theta) &= (1-\theta)\left[m(1, n+1, N, \theta) - m(\sigma, n, N, \theta)\right] \\
&\quad \times \left[m(1, n+1, N, \theta) + m(\sigma, n, N, \theta)\right] \\
&= a(n, N)\left(\frac{N}{1-\theta} - 1\right)\left[2 - b(n, N)(N - (1-\theta))\right], \quad (32)
\end{aligned}$$

$$\text{with } a(n, N) \equiv \frac{\kappa}{N - (n+1)\kappa}\left(1 - \frac{1}{N - n\kappa}\right) > 0,$$

$$\text{and } b(n, N) \equiv \frac{1 - \kappa}{N - (n+1)\kappa} + \frac{1}{N - n\kappa} > 0.$$

We can then get, for the sign of the elasticity of G with respect to θ,

$$\text{sign}\left\{\epsilon_\theta G(n, N, \theta)\right\} = \text{sign}\left\{(1-\theta)^2 - N^2\left(1 - \frac{2}{Nb(n, N)}\right)\right\}. \quad (33)$$

For $\theta \in \left[\theta^L(n+1), \theta^L(n)\right]$, we have:

$$G(n, N, \theta) = (1 - \theta)\left(\left(\frac{1}{n+1}\right)^2 - \left(\frac{1 - n\kappa/(1-\theta)}{N - n\kappa}\right)^2\right), \quad (34)$$

with sign of its elasticity with respect to θ

$$\text{sign}\left\{\epsilon_\theta G(n, N, \theta)\right\} = \text{sign}\left\{1 - \left(\frac{N - n\kappa}{n+1}\right)^2 - \left(\frac{n\kappa}{1-\theta}\right)^2\right\}. \quad (35)$$

Finally, for $\theta \in \left[\theta^L(n), 1\right]$,

$$G(n, N, \theta) = \frac{1-\theta}{(n+1)^2}. \quad (36)$$

Looking at equations (33) and (35), we see that the elasticity of G can change signs at most once, from positive to negative, in each one of the two first intervals. Also, since by (16) and (17) the partial derivative of $m(1, n+1, N, \theta)$ with respect to θ switches from positive to nil at $\theta = \theta^L(n+1)$, and the corresponding partial derivative of $m(\sigma, n, N, \theta)$ is continuous at the same point, we see from (18) and (19) that the right-hand partial derivative of G with respect to θ at $\theta^L(n+1)$ must be smaller than the corresponding left-hand derivative. Hence, G is strictly quasi-concave in θ when restricted to the interval $\left[0, \theta^L(n)\right]$. As G is clearly decreasing in θ in the interval $\left[\theta^L(n), 1\right]$, we can conclude that G is in fact strictly quasi-concave in θ. We may add that G is never monotonically increasing, since the interval $\left[\theta^L(n), 1\right]$ is non-degenerate for $n > 0$ and, by (35), G is otherwise decreasing in the interval $\left[\theta^L(n+1), \theta^L(n)\right] = \left[\theta^L(1), 1\right]$.

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Now let us consider the case where either the number n of successful firms or their relative cost advantage κ is small, so that $\theta^L(n+1) > 0$. By (32) and (33), the function G is increasing for θ close to zero if and only if

$$Nb(n,N) - 2 = \left(\frac{n}{N - n\kappa} - \frac{N - (n+1)}{N - (n+1)\kappa}\right)\kappa < \frac{2}{N^2 - 1}. \qquad (37)$$

A simple inspection shows that this inequality holds either for κ close to zero, or for $n = 0$, and is violated if both κ and n are large enough.

It remains to show that the incremental gain G is a decreasing function of n when κ is small enough. Using again (16), (17), (18) and (19), we obtain for $\theta \in \left[0, \theta^L(n+1)\right]$:

$$
\begin{aligned}
G(n, N, \theta) &= \left[\left(\frac{1}{N - 1 + \theta} - \frac{1 - \kappa}{N - \kappa - n\kappa}\right)^2 - \left(\frac{1}{N - 1 + \theta} - \frac{1}{N - n\kappa}\right)^2\right] \\
&\quad \times \frac{(N - 1 + \theta)^2}{1 - \theta}. \qquad (38)
\end{aligned}
$$

We see that the terms $(1 - \kappa)/(N - \kappa - n\kappa)$ and $1/(N - n\kappa)$ have both positive elasticities, the elasticity of the former term being the larger one. As a consequence, the first square within the brackets decreases faster than the second as n increases, so that G is indeed decreasing in n. Also, for $\theta \in \left[\theta^L(n), 1\right]$, $G(n, N, \theta) = (1 - \theta)/(n + 1)^2$, and is clearly decreasing in n. For $\theta \in \left[\theta^L(n+1), \theta^L(n)\right]$, the two squares in (34) are both decreasing in n, but the elasticity of the second one is smaller in absolute value, leading to the same conclusion, for κ small enough.

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