Why Limiting Price or Supply?

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Abstract

This paper analyzes the role of seller-induced excess demand as a signal of high quality of a new indivisible product provided by a monopoly firm. We show that creating shortages by limiting supply can signal high quality and result in more profitable separating equilibria. Our results provide a rationale for the understanding of why high quality producers may prefer to limit supplies (e.g. queuing, limited edition).

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1 Introduction

Signaling the existing quality of an experienced good is of vital importance to firms. Beginning with Nelson (1970, 1974), the signaling role for advertising has received considerable attention in the literature. A basic idea is that advertising may be *dissipative*, in the sense that it is only a signal that the firm is able to spend a lot of money, but consumers can observe the total amount of money or a proxy of it that the firm is spending on advertising. It is therefore possible to have an equilibrium in which consumers rationally expect the firm to spend different amounts on

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advertising for different product quality types. Signaling quality through advertising, however, cannot help to explain the puzzling phenomenon of why a seller of a new product which is scarce does not increase the introductory price or supply to eliminate scarcity. The phenomenon is commonly observed in many situations.

There are several papers study the seller induced shortages (or excess demand). In DeGraba (1995), excess demand is purposely induced by a monopoly firm to promote buying frenzy. In his model, consumers learn their types over time. By selling fewer units, uncertainties will be created for those consumers to face who choose to wait till their have leaned their types in the second period, in the sense that there will be no enough supply. Consequently, all consumers purchase the good while uninformed although they prefer to purchase after become informed. This induced buying frenzy allows the monopolist to price higher and earn greater profit.

Allen and Faulhaber (1991) argue that if *Nelson effect* dominates *Schemalensee effect*, then the high quality firm uses low price to signal its quality. It is exogenously assumed that there are not enough good firms to meet all market demand[1]. In their separating equilibrium where good firms are preferred by consumers, rationing naturally occurs and consumers are willing to pay more than the low (signalling) price set by the good firms. Nevertheless, the good firms must keep the price low to maintain its signalling value despite the fact that consumers faced with rationing have an incentive to make a side-payment. Although this research is highly related to the current one, there are significant differences. First, the shortage is exogenous in Allen and Faulhaber (1991) where in our model it is endogeneous. The results are also different. In Allen and Faulbaher, the signalling price is always the low price where in our model it can be the consumers' highest willingness-to-pay. In other words, the signal is conveyed by price only in Allen and Faulhaber but by price-shortage pair in our model.

In a related paper, Wilson (1980) identifies an equilibrium where there is an excess supply accompanied with high price. This is because higher market price induces high quality car owners (who have higher reservation value) to bring their cars to the market hence creating excess supply. Despite the fact that there is an

[1] This is because if there were enough good firms to meet all demand, then in competitive equilibrium good firms would compete the price down to a level that bad firms would quit. Hence signalling problem disappears.

excess supply, buyers are not willing to lower the price because high quality cars will quit the market if otherwise, leaving market flooded by lemons only.[2]

In industries other than restaurant, limiting supply is a commonly used business strategy. For instance, the high quality handbag producer Louis Vuitton does not advertise against a cheap handbag imitation producer but frequently produces limited edition bags. Apart from limited editions, some firms also limit the supply of a new product in its debut. In a recent example, the automaker Jaguar puts only 50 copies of a special Neiman Marcus edition of the redesigned 2010 Jaguar 5.0-liter V8 making 470 hp XJ at a price of $105,000 for order in October 2009. But, it only took four hours and four minutes to finish all bookings. The dealers then will still take orders and compile a waiting list, and the car will be on sale nationwide in US after being revealed in July 2010 (*Autoweek*, October 20, 2009). Ferrari promises that it will not produce more than 4300 vehicles despite more than a two-year waiting list for its cars (*Financial Times*, April 11, 2002). It is reported that the owner of the Italian brand Armani who has been steadily profitable for over 30 years, Mr. Giorgio Armani, said: "... they should be made more exclusive by restricting sales" (*Economist*, October 2-8, 2004).

In this paper, we demonstrate that limiting supply initially can be a less costly way to signal quality. The basic idea is that consumers can observe the shortage purposely created by the firm by proxies such as the time required to order in advance or queuing. They can then update their beliefs about the quality type using the observed shortage. To best illustrate the idea, we confine analysis to a two-period discrete model of a monopoly firm providing a new product of either high or low quality. The firm knows the quality but the consumers do not have this knowledge at the time the product is introduced. Further, the firm cannot vary the quality.[3] The two period model captures the introductory phase of the product under the following conditions on information transmission. The product's life cycle can be decomposed into two phases: the introductory phase and the mature phase. Signaling occurs during the introductory phase. We do not allow word-

[2]Bose (1996) and von Ungern-Sternberg (1991) consider the rationing problem in restaurants but neither discuss signaling role of rationing. Bose (1996) claims that restaurants using capacity and henceforth queuing to screen less profitable customers since serious customers who will spend more care less about waiting time. von Ungern-Sternberg's argument is somewhat similar to peak load pricing: since restaurants can not charge the time customers spend in dining so excess demand is associated with profit maximizing price.

[3]As explained in Milgrom and Roberts (1986), this may correspond to the situation where the firm's R&D effort has generated the product of a given quality that the firm must then decide how to introduce. As usual, we assume that the product of high-quality is more costly to produce.

of-mouth learning or independent sources of quality revelation such as consumer reports. However all consumers know the product quality in the mature phase either through a separating equilibrium where consumers can rationally predict the true quality, or in a pooling equilibrium where all consumers buy in the introductory period and know the true quality from their experience. Hence the firm will choose its complete-information monopoly price and will not induce excess demand in the mature phase.

The rest of the paper is organized as follows. Section 2 introduces the model in the simple homogenous consumer case. Section 3 presents results on heterogeneous consumer case. Section 4 concludes the paper.

2 Quality Signaling Problem

Consider a market for a new indivisible experience good supplied by a monopoly firm with two periods, period 1 and period 2. The good can be of either high (type s_1) or low quality (type s_0) with $s_0 < s_1$. The quality is not observable to the consumers nor is adjustable by the firm across time.[4] The marginal cost of the firm is constant for either quality type and is denoted by c_t for type s_t. Assume as usual $c_0 < c_1$.

There is a continuum of consumers with each demanding for at most one unit of the good in each period. Consumers may or may not be homogenous with regard to their tastes or willingness to pay for quality. Let θ denote the value of quality for a consumer, so that if he consumes the good of quality s_k at price p, then he obtains utility

$$\theta s_k - p. \tag{1}$$

Let $\delta \in (0, 1]$ be the common discount factor and let μ be the probability with which each consumers initially believes that the monopolist's product has high quality. Since $s_1 > s_0$ and $c_0 < c_1$, the low-quality type has incentives to mislead the consumers into believing that it is of the high-quality type, provided that it is not too costly to do so.[5] Consumers are price-taking which is automatic for the present setting.

[4] As explained in Milgrom and Roberts (1986), this may correspond to the situation, where the firm's R&D effort has generated the product of a given quality that the firm must then decide how to introduce.

[5] Since technology is not adjustable, we can identify the type of the firm with the quality type of the good it produces.

There is no communication between the consumers. Therefore, those who purchase the good in the first period learn the quality and can make their second period purchase decisions based on that information. The consumers who do not purchase will base their period 2 decisions upon beliefs updated using observable signals.[6]

3 Quality Signaling with Homogenous Consumers

In this section, we consider the simpler case in which consumers are homogenous, in the sense that they have the same taste for quality. We show that for the high-quality firm, using price and advertising to signal high quality is less profitable than not to signal. In case when the low-quality product is also socially desirable under complete information, the high-quality firm cannot signal its quality with price and advertising. The reason why price alone or price-advertising combination fails to signal high quality is that the high quality firm's choice can be always profitably mimicked by the low-quality type, and there is no gain in the future by doing so. The result is discussed in Tirole (1988) in which first period price is uninformative if the low-quality monopolist can earn profit under full information. However, it is shown in this paper that limiting supply (inducing shortage) is more costly to the low-quality firm since the opportunity cost in terms of the lost profit is higher due to cost differential $c_0 < c_1$. Consequently, a suitable price together with a certain amount of shortage generated from limited supply can more profitably signal the high quality.

3.1 Signal High quality by Price and Advertising

Suppose that the high-quality firm uses price p_1 and dissipative advertising with expenditure A to signal its quality. As usual, in any separating equilibrium, the low-quality firm chooses its complete information monopoly price θs_0 and does not advertise. Hence, the following incentive-compatibility constraints characterize all

[6] Strangers to a tourism town often find themselves unable to judge the quality of local restaurants even if surrounded by advertisements.

separating equilibria:[7]

$$p_1 - A - c_1 \geq \theta s_0 - c_1 \tag{2}$$

and

$$p_1 - A - c_0 \leq \theta s_0 - c_0. \tag{3}$$

Notice that (2) and (3) imply $p_1 - A = \theta s_0$. Thus, the net price of the high-quality firm is the same as the low-quality firm's price. In contrast, the common price for both types of the firm in the pooling equilibrium is given by:

$$p_1^{\mu} = \mu \theta s_1 + (1 - \mu)\, \theta s_0 > \theta s_0.$$

This period 1 price results in the high-quality firm's total profit across the two periods equal to

$$p_1^{\mu} - c_1 + \delta(\theta s_1 - c_1). \tag{4}$$

Since $s_0 < s_1$, it follows that the polling equilibrium is more profitable than the separating equilibrium with price and dissipative advertising as the signal of high quality.

3.2 Signaling Quality by Limiting Supply

Suppose that, instead of supplying the entire population of consumers, the high-quality firm limits its supply so that only a fraction $\alpha \in (0, 1)$ of the population of consumers can get the good. That is, suppose that the high-quality firm induces a shortage equal to the amount of $(1 - \alpha)$. The high-quality firm may reveal its type to the consumers by properly choosing the amount of shortage, leaving it not desirable for the low-quality type to mimic.

In a separating equilibrium, the low-quality type does not induce any shortage because it cannot mislead the consumers unless it induces the same or larger amount of shortage. This means that all separating equilibria are equally profitable for the low-quality firm. However, this is not true for the high-quality firm. We are interested in the most profitable separating equilibria for the reason that they satisfy refinements such as the Cho-Kreps intuitive criterion. Such equilibria are known

[7] Let (p_1, A) satisfy (1) and (2), and let $\mu(h|p_1', A')$ denote the probability with which each consumer believes that the good if of high quality conditional on observing price p_1 and advertisement measure by expenditure A'. Then, (p_1, A) and $(\theta s_0, 0)$ can be supported as the firm's equilibrium choices by letting $\mu(h|p_1', A') = 1$ for all pairs $(p_1', A') \geq (p_1, A)$ and $\mu(h|p_1', A') = 0$ otherwise. Notice that such a belief system in turn is consistent with Bayes rule and the firm's equilibrium strategy.

as “least-cost" separating equilibria. A price-shortage pair (p^*, α^*) is a least-cost separating equilibrium if and only if it solves

$$\max_{\alpha, p_1 \leq \theta s_1} \alpha \left(p_1 - c_1\right) + \delta \left(\theta s_1 - c_1\right) \tag{5}$$

subject to

$$\alpha(p_1 - c_1) \geq \theta s_0 - c_1, \tag{6}$$

and

$$\alpha(p_1 - c_0) + \delta \max\{\theta s_0 - c_0, (1-\alpha)(\theta s_1 - c_0)\} \leq (1+\delta)(\theta s_0 - c_0). \tag{7}$$

Condition (6) is equivalent to the incentive constraint

$$\alpha(p_1 - c_1) + \delta(\theta s_1 - c_1) \geq \theta s_0 - c_1 + \delta(\theta s_1 - c_1),$$

under which the high-quality firm does not have any incentive to mimic the low-quality type's period 1 choice of $(\theta s_0, 0)$. Condition (7) is the incentive constraint which makes it not desirable for the low-quality firm to mimic the high-quality firm's period 1 choice of (p, α), and then either supplies to the entire population of consumers at price θs_0 or supplies only to $(1-\alpha)$ fraction, who did not consume the good in period 1, by continuing to mimic the high-quality firm's choice of charging price θs_1 in period 2.[8]

Lemma 1 *Let (p_1^*, α^*) be the period 1 choice of the high-quality firm in a least-cost separating equilibrium. Then,*

$$\alpha(p_1 - c_0) + \delta \max\{\theta s_0 - c_0, (1-\alpha)(\theta s_1 - c_0)\} = (1+\delta)(\theta s_0 - c_0). \tag{8}$$

Proof. Let (p_1, α) be a pair such that

$$\alpha(p_1 - c_0) + \delta \max\{\theta s_0 - c_0, (1-\alpha)(\theta s_1 - c_0)\} < (1+\delta)(\theta s_0 - c_0) \tag{9}$$

Suppose first $\theta s_0 - c_0 \geq (1-\alpha)(\theta s_1 - c_0)$. In this case, if $p_1 < \theta s_1$, then, there exists a price from $p_1' > p_1$ such that (p_1', α) also satisfies (9). Since $\alpha(p_1' - c_0) > \alpha(p_1 - c_0)$, (p_1', α) satisfies (6) whenever (p_1, α) does. Since (p_1', α) is more profitable than (p_1, α), the latter cannot solve problem (5). If $p_1 = \theta s_1$, then (8) reduces to $\alpha(p_1 - c_0) < \theta s_0 - c_0$. Hence, $\alpha < 1$ because $\theta s_1 - c_0 > \theta s_0 - c_0$. It follows that,

[8]When mimicking in period 1, the low-quality firm can only mislead consumers who did not buy in period 1.

by slightly increasing α to $\alpha' > \alpha$, we can guarantee that (p_1, α') also satisfies (9).[9] Since $\alpha'(p_1 - c_1) > \alpha(p_1 - c_1)$, (p_1, α) cannot solve problem (5).

Suppose now $\theta s_0 - c_0 < (1-\alpha)(\theta s_1 - c_0)$ which implies $\alpha \neq 1$. Thus, as before, we can increase the maximum value of problem (5) by keeping price p_1 while slightly increasing α without violating (6) and (7). ■

Lemma 1 shows that (7) must be binding at any solution to problem (5) or equivalently, in a least-cost separating equilibrium. We now apply this lemma to show that there is a unique solution for problem (5) at which there is a positive shortage under the following conditions.

A1:

$$\delta(\theta s_1 - c_0) > c_1 - c_0,\ \theta(s_1 - s_0) > \theta s_0 - c_0,\ \theta s_k > c_k,\ k = 0, 1.$$

The intuition for the first inequality is that discounting has to be large enough relative to the ratio of the cost differential $c_1 - c_0$ to the profit the low-quality firm gets from a consumer when it is perceived as the high-quality firm. With the second inequality, the value differential due to quality difference exceeds the profit the low-quality firm gets under complete information. The third inequality is self-explanatory.

Proposition 1 *Assume **A1**. Then, there is a unique separating equilibrium in which*

$$\alpha^* = \frac{\theta(s_1 - s_0)}{\theta s_1 - c_0}$$

and

$$p_1^* = c_0 + \frac{(\theta s_0 - c_0)(\theta s_1 - c_0)}{\theta(s_1 - s_0)}.$$

Proof. Let (p_1, α) be a solution for problem (5). We break the rest of the proof into two cases.

- **Case 1:** $\alpha \geq \theta(s_1 - s_0)/(\theta s_1 - c_0)$.

In this case, we have $(\theta s_0 - c_0) \geq (1-\alpha)(\theta s_1 - c_0)$.[10] Thus, by (9), $\alpha(p_1 - c_0) = (\theta s_0 - c_0)$, which implies $\alpha p_1 = \alpha c_0 + (\theta s_0 - c_0)$. It follows that $\alpha(p_1 - c_1) = \theta s_0 - c_0 - \alpha(c_1 - c_0)$ is decreasing in α.

[9] Notice $\theta s_0 - c_0 \geq (1-\alpha)(\theta s_1 - c_0)$ implies $\theta s_0 - c_0 > (1-\alpha')(\theta s_1 - c_0)$ for all $\alpha' > \alpha$.

[10] Notice $\theta s_0 - c_0 \geq (1-\alpha)(\theta s_1 - c_0)$ if and only if $\alpha(\theta s_1 - c_0) \geq \theta(s_1 - s_0)$ or equivalently $\alpha \geq \theta(s_1 - s_0)/(\theta s_1 - c_0)$.

- **Case 2:** $\alpha \leq \theta(s_1 - s_0)/(\theta s_1 - c_0)$.

In this case, we have $(\theta s_0 - c_0) \leq (1-\alpha)(\theta s_1 - c_0)$. Thus, by (9) in Lemma 1, $\alpha(p_1 - c_0) + \delta(1-\alpha)(\theta s_1 - c_0) = (1+\delta)(\theta s_0 - c_0)$ which implies

$$\alpha p_1 = \alpha c_0 - \delta(\theta s_1 - c_0)(1-\alpha) + (\theta s_0 - c_0)(1+\delta).$$

It follows that $\alpha(p_1 - c_1) = (1+\delta)(\theta s_0 - c_0) - \delta(\theta s_1 - c_0) + \delta\alpha(\theta s_1 - c_0) - \alpha(c_1 - c_0)$. Hence, by **A1**, $\alpha(p_1 - c_1)$ is increasing in α.

In summary, we have shown that $\alpha(p_1 - c_1)$ is decreasing in α when $\alpha \geq \theta(s_1 - s_0)/(\theta s_1 - c_0)$ and increasing in α when $\alpha \leq \theta(s_1 - s_0)/(\theta s_1 - c_0)$. This concludes that in any least-cost separating equilibrium it must be

$$\alpha = \frac{\theta(s_1 - s_0)}{\theta s_1 - c_0}$$

and

$$p_1 = c_0 + \frac{\theta s_0 - c_0}{\alpha} = c_0 + \frac{(\theta s_0 - c_0)(\theta s_1 - c_0)}{\theta(s_1 - s_0)}.$$

This establishes both the uniqueness and characterization of the solution for problem (5).

To show the existence, notice first that by **A1**, the price-shortage pair (p^*, α^*) in the proposition satisfies $0 < \alpha^* < 1$ and $p_1^* < \theta s_1$ (**Need to verify that the total profit across the two periods is non-negative. By (10), this seems to require** $(1+\delta)\theta s_0 \geq c_0 + \delta c_1$**.** I don't get a simple condition like this, I cannot simplify (10) much). By the preceding analysis, (p_1^*, α^*) maximizes $\alpha(p_1 - c_1)$ subject to (7). Thus, to complete the rest of the proof, it suffices to show that (p_1^*, α^*) also satisfies (6). To this end, notice that

$$\alpha^*(p_1^* - c_1) \geq (\theta s_0 - c_1) \Longleftrightarrow \theta(s_1 - s_0) \leq s_1 - c_0.$$

Since $\theta s_0 > c_0$, the above condition is automatically satisfied. ■

By Proposition 1, the high-quality firm's total profit across the two periods in the least-cost separating equilibrium is:

$$\alpha^*(p_1^* - c_1) + \delta(\theta s_1 - c_1) = \theta s_0 - c_0 + \delta(\theta s_1 - c_1) - \frac{\theta(s_1 - s_0)}{\theta s_1 - c_0}(c_1 - c_0). \quad (10)$$

From (4) and (10) it follows that the high-quality firm is better off signaling its quality type via the price-shortage pair if and only if $p_1^\mu - c_1 < \alpha^*(p_1^* - c_1)$, which in turn is equivalent to

$$\mu < \frac{(\theta s_0 - c_0)(c_1 - c_0)}{\theta(s_1 - s_0)(\theta s_1 - c_0)}. \tag{11}$$

We summarize this result in the following proposition whose proof will be omitted.

Proposition 2 *Assume* ***A1****. Then, the price-shortage combination is a more profitable signal than the price-advertising combination if and only if μ satisfies (11).*

4 Signalling Quality with Heterogeneous Consumers

In this section we consider a familiar generalization of the model in Section 3 that allows for heterogeneous consumers. We follow Wolinsky (1983), Chan and Leland (1982), Cooper and Ross (1984, 1985), Farrell (1980), and Tirole (1988), among several others to consider unit-demand differentiated consumer. We assume that there are two types of consumers in terms of their tastes for quality. The value of the good with quality level s_k is $\theta_1 s_k$ for type 1 consumers and $\theta_0 s_k$ for type 0 consumers. The proportion of type 1 consumers is denoted by q_1. As with only homogenous consumers, price and advertisement are substitutes for the high-quality firm. Due to heterogeneity, our assumption **A1′** changes to

A1′:

$$\delta(\theta_1 s_1 - c_0) > c_1 - c_0,\ \theta_1 s_1 - \theta_0 s_0 > \theta_0 s_0 - c_0,\ \theta s_k > c_k,\ k = 0, 1.$$

Unlike the homogenous case, we show that the high-quality firm is better off signaling its quality type via price alone than not signaling at all. Furthermore, for a certain range of the parameter values for the model, the high-quality firm can profitably signal its quality by limiting its supply than by price alone.

We make the following further assumptions:

A2:

$$\frac{\theta_0 s_0 - c_0}{\theta_1 s_1 - c_0} \leq q_1 \leq \frac{\theta_0 s_0 - c_0}{\theta_1 s_0 - c_0}.$$

Denote by $\underline{q}_1$ the lower bound and $\overline{q}_1$ the upper bound in the **A2**. This assumption implies that it is profitable for the low-quality firm to sell its good to both types of consumers under full information. In addition, it also makes the quality signaling problem non-trivial, because it implies that the low-quality firm is better off charging the high-quality firm's complete information monopoly price, if in so doing it is perceived as the high-quality firm. Notice also that **A2** implies

$$q_1 > \frac{\theta_0 s_0 - c_1}{\theta_1 s_1 - c_1}.$$

This means that when information is complete, it is more profitable for the high-quality firm to sell its good to type 1 consumers only at price $\theta_1 s_1$ than to sell to both types of consumers at price $\theta_0 s_0$.

4.1 Signaling Quality by Price

When the high-quality firm uses price to signal its quality type, the incentive compatibility constraints become

$$q_1 \left[(p_1 - c_1) + \delta \left(\theta_1 s_1 - c_1 \right) \right] \geq \theta_0 s_0 - c_1 + \delta q_1 \left(\theta_1 s_1 - c_1 \right), \tag{12}$$

and

$$q_1 \left(p_1 - c_0 \right) + \delta \left(\theta_0 s_0 - c_0 \right) \leq \left(1 + \delta \right) \left(\theta_0 s_0 - c_0 \right). \tag{13}$$

Observe that (12) and (13) are consistent under assumption **A2**, in the sense that there are prices that simultaneously satisfy them. Observe also that in least-cost separating equilibrium, the high-quality firm's price solves

$$\begin{aligned} &\max_{p_1 \leq \theta_1 s_1} q_1 \left[(p_1 - c_1) + \delta \left(\theta_1 s_1 - c_1 \right) \right] \\ &\text{subject to (12) and (13).} \end{aligned} \tag{14}$$

Simple analysis shows that the high-quality firm's price in the least-cost separating equilibrium must be:

$$p_1^* = c_0 + \frac{\theta_0 s_0 - c_0}{q_1}. \tag{15}$$

For later references, we summarize this result in the following proposition whose proof will be omitted.

Proposition 3 *Assume **A2**. Then, there exists a unique least-cost separating equilibrium, in which the high-quality firm signals its quality type in period 1 by price p_1^* in (15).*

The following example provides an illustration of the model with both **A1** and **A2** satisfied.

Example 1: Let $c_1 = \frac{1}{2}, c_0 = 0, \theta_1 = 2, \theta_0 = 1, s_1 = 1, s_0 = \frac{1}{2}, \delta = 1$. Then, **A1′** and **A2** are satisfied with $\left[\frac{1}{4}, \frac{1}{2}\right]$ as the interval of feasible proportions q_1. The least-cost separating signalling price is monotonically decreasing with respect to the proportion of high-value consumers from $p_1^* = 2$ when $q_1 = \frac{1}{4}$ and $p_1^* = 1$ when $q_1 = \frac{1}{2}$ (see Tirole, 1988, pp. 121, exercise 2.7 for further details).■

4.2 Signaling High Quality by Limiting Supply

Since a consumer cannot learn the quality of the good if he does not consume in period 1, the incentive compatibility constraints when the high-quality firm signals its quality type by limiting supply become

$$\alpha q_1 (p_1 - c_1) + \delta q_1 (\theta_1 s_1 - c_1) \geq \theta_0 s_0 - c_1 + \delta q_1 (\theta_1 s_1 - c_1) \tag{16}$$

and

$$\alpha q_1 (p_1 - c_0) + \delta \max \{\theta_0 s_0 - c_0, (1 - \alpha) q_1 (\theta_1 s_1 - c_0)\} \leq (1 + \delta) (\theta_0 s_0 - c_0). \tag{17}$$

Constraint (16) means that the high-quality firm prefers to supply only to the high taste consumers with shortages. Constraint (17) ensures that low-quality firm does not want to mislead or mimic the high-quality type's strategy.

In a least-cost separating equilibrium, the high-quality firm's period 1 choice (p_1, α) solves

$$\begin{aligned} &\max_{p_1 \leq \theta_1 s_1} \alpha q_1 [(p_1 - c_1) + \delta (\theta_1 s_1 - c_1)] \\ &\text{subject to (16) and (17).} \end{aligned} \tag{18}$$

As with the previous case, the incentive compatibility constraint (17) is binding in any least-cost separating equilibrium. We summarize this result in the following lemma. Its proof is similar to the proof of Lemma 1. For this reason, we omit the proof.

Lemma 2 *Let (p_1, α) be the high-quality firm's period 1 choice in a least-cost separating equilibrium. Then, (17) must be binding.*

In what follows we characterize the unique least-cost separating equilibrium separately for two disjoint ranges of parameter values. The first range is determined by the following assumptions:

A3:

$$q_1(\theta_1 s_1 - c_0) \geq 2(\theta_0 s_0 - c_0).$$

Denote the lower bound in **A3** by $\underline{q}_1$. The following proposition establishes a unique least-cost separating equilibrium under assumptions **A1′, A2** and **A3**.

Proposition 4 *Assume **A1′**, **A2**, and **A3**. Then, there exists a unique least-cost separating equilibrium in which the high-quality firm chooses*

$$\alpha^* = 1 - \frac{\theta_0 s_0 - c_0}{q_1 (\theta_1 s_1 - c_0)}, \tag{19}$$

and

$$p_1^* = c_0 + \frac{(\theta_1 s_1 - c_0)(\theta_0 s_0 - c_0)}{q_1 (\theta_1 s_1 - c_0) - (\theta_0 s_0 - c_0)}. \tag{20}$$

Proof. The proof is similar to that of Proposition 1. We present it in the appendix. ■

Example 2: Consider the same parameter values for $\theta_1, \theta_0, s_1, s_0, c_1, c_0, \delta$ as in Example 1. When $q_1 = \frac{1}{2}$, the high-quality firm's first period profit in the least-cost separating equilibrium is $\frac{3}{8}$ when signaling with limited supply; its profit is $\frac{1}{2}$ when signaling by price only. Thus, limited supply is a more profitable signal than price ■

Depending on consumers' prior belief, the least-cost separating equilibrium in Proposition 3 may be less profitable than the pooling equilibrium. Thus, to guarantee that the high-quality firm has incentive to signal quality by limiting supply, we need the following condition on the prior belief.

$$\mu \geq \frac{(1-\alpha^*)(c_1 - c_0) - s_0(\theta_1 - \theta_0)}{\theta_1(s_1 - s_0)}.$$

(We need to simplify the above expression? Should we include it in A4 to make it more formal? My opinion is to leave it here. Because this is only a condition to ensure separating equilibrium. If we include this to A4 then in homogeneous case we also need to do so. It is not necessary. Everyone knows conditions on priors are required to have separating.)

We now consider the other range of parameter values guaranteeing the existence of a least-cost separating equilibrium. This range is determined by

A4:

$$\frac{\theta_0 s_0 - c_0}{\theta_1 s_1 - c_0} < q_1 \le \frac{2(\theta_0 s_0 - c_0)}{\theta_1 s_1 - c_0}.$$

Notice that the price in (20) is not feasible under **A4**. We show that the high-quality firm must set the highest price in least-cost separating equilibrium.

Proposition 5 *Assume* ***A1'*** *and* ***A4****. Then, there is a unique least-cost separating equilibrium in which the high-quality firm's signalling strategy is given by*

$$\alpha^* = \frac{\theta_0 s_0 - c_0}{q_1 (\theta_1 s_1 - c_0)}, \tag{21}$$

and

$$p_1^* = \theta_1 s_1. \tag{22}$$

Proof. By Lemma 2, (17) must be binding in least-cost separating equilibrium:

$$\alpha q_1(p_1 - c_0) + \delta \max\{\theta_0 s_0 - c_0, (1-\alpha) q_1(\theta_1 s_1 - c_0)\} = (1+\delta)(\theta_0 s_0 - c_0). \tag{23}$$

Suppose first $\theta_0 s_0 - c_0 < (1-\alpha) q_1(\theta_1 s_1 - c_0)$. This together with (23) imply

$$\alpha < 1 - \frac{\theta_0 s_0 - c_0}{q_1(\theta_1 s_1 - c_0)}, \; p_1 = c_0 + \frac{(1+\delta)(\theta_0 s_0 - c_0) - \delta(1-\alpha) q_1(\theta_1 s_1 - c_0)}{\alpha q_1}. \tag{24}$$

The total profit of the high-quality firm is increasing in α within the range in (24) because

$$\Big\{ q_1[\alpha(p_1 - c_1) + \delta(\theta_1 s_1 - c_1)] \Big\}' = q_1[\delta(\theta_1 s_1 - c_0) - (c_1 - c_0)] > 0.$$

On the other hand,

$$\alpha = 1 - \frac{\theta_0 s_0 - c_0}{q_1(\theta_1 s_1 - c_0)} \Rightarrow p_1 = c_0 + \frac{\theta_0 s_0 - c_0}{\alpha q_1} > \theta_1 s_1.$$

Thus, the optimal pair is $(\tilde{p}_1, \tilde{\alpha})$ with $\tilde{p}_1 = \theta_1 s_1$ and

$$\tilde{\alpha} = \frac{1}{q_1}\left[\left(\frac{1+\delta}{1-\delta}\right)\left(\frac{\theta_0 s_0 - c_0}{\theta_1 s_1 - c_0}\right) - \frac{\delta}{1-\delta} \right].$$

On the other hand,

Suppose now $\theta_0 s_0 - c_0 \geq (1-\alpha) q_1(\theta_1 s_1 - c_0)$. This together with (23) implies

$$\alpha \geq 1 - \frac{\theta_0 s_0 - c_0}{q_1\left(\theta_1 s_1 - c_0\right)} \text{ and } p_1 = c_0 + \frac{(\theta_0 s_0 - c_0)}{\alpha q_1}. \tag{25}$$

In this case,

$$\left\{ q_1[\alpha(p_1 - c_1) + \delta(\theta_1 s_1 - c_1)] \right\}' = -q_1(c_1 - c_0) < 0.$$

Hence, the total profit is decreasing in α within the range in (25). On the other hand,

$$\alpha = 1 - \frac{\theta_0 s_0 - c_0}{q_1(\theta_1 s_1 - c_0)} \Rightarrow p_1 = c_0 + \frac{\theta_0 s_0 - c_0}{q_1 - \frac{\theta_0 s_0 - c_0}{\theta_1 s_1 - c_0}} > \theta_1 s_1.$$

Thus, the optimal pair is $(\hat{p}_1, \hat{\alpha})$ with $\hat{p}_1 = \theta_1 s_1$ and

$$\hat{\alpha} = \frac{\theta_0 s_0 - c_0}{q_1(\theta_1 s_1 - c_0)}.$$

With $p_1 = \theta_1 s_1$, the high-quality firm's profit is monotonically increasing in α. Since

$$\hat{\alpha} > \tilde{\alpha} \Leftrightarrow \theta_1 s_1 - \theta_0 s_0 > \theta_0 s_0 - c_0,$$

the optimal choice of shortage is such that $\alpha = \theta_0 s_0 - c_0 / q_1(\theta_1 s_2 - c_0)$. Finally, notice that with these values for p and α, the pair (p, α) satisfies the incentive constraint (15) for the high-quality firm if and only if

$$\frac{\theta_0 s_0 - c_0}{\theta_1 s_1 - c_0} \geq \frac{\theta_0 s_0 - c_1}{\theta_1 s_1 - c_1}.$$

The preceding inequality automatically holds because $c_1 > c_0$.[11] ■

4.3 Signal Comparisons

In what follows, we characterize the existence and comparisons of separating equilibria (**What kind?** BY PROPOSITIONS 3, 4, AND 5) as well as pooling equilibria (**What kind?** WITH DIFFERENT μ) when consumers differ(**?** ARE HETER0GENOUS.). The following assumption guarantees that high-quality period 1 price in the least cost separating equilibrium when signaling with limited supply strictly below the highest possible price with complete information.

[11] Given two constants a and b, the ratio $(a-x)/(b-x)$ is decreasing in x if and only if $a < b$.

A5:

$$\theta_1 (s_1 - s_0) > \theta_1 s_0 - c_0.$$

Assumption A5 guarantees that $\widetilde{q}_1 < \overline{q}_1$ hence not all separating equilibria characterized by Proposition 4 are dominated (**Not clear. DO we mean for all q_1?** I MEAN THAT IF A5 HOLDS, THEN SOME SEPARATING WITH PRICE-SHORTAGE PAIRS WILL ARISE IN EQUILIBRIUM, INSTEAD OF ALL BY PRICE ALONE. I PROVIDE A FIGURE IN PDF).

Corollary 1. *Assume* **A5** *holds (**Don't we need the other assumptions?**). then separating equilibria characterized by Proposition 4 in which the monopoly firm signal through a price-shortage pair with highest possible price are always most profitable separating equilibria for $q_1 \in [\widehat{q}_1, \overline{q}_1]$.*

Corollary 2. *Assume* **A2** *holds. then separating equilibria exist (**What separating equilibrium?** IN PROPOSITION 3). Let*

$$\widehat{q}_1 \equiv \frac{(\theta_0 s_0 - c_0)(\theta_1 s_1 - c_1)}{(\theta_1 s_1 - c_0)(c_1 - c_0)}.$$

Then if

$$c_1 - c_0 \leq \theta_1 s_1 - c_1 \leq 2 (c_1 - c_0)$$

holds, then equilibria characterized by (15) using price alone to signal is the most profitable separating equilibira for $q_1 \in \left[\underline{q}_1,\ \widehat{q}_1\right]$; equilibria characterized by Proposition 5 in which firm signals through a price-shortage pair with highest possible price is the most profitable separating equilibria for $q_1 \in [\widehat{q}_1, \widetilde{q}_1]$. **Do we need a proof here? Do we need to list scenarios when part or all of $c_1 - c_0 \leq \theta_1 s_1 - c_1 \leq 2 (c_1 - c_0)$ fails?** I DON'T THINK WE NEED TO LIST ALL, THAT COMPLICATES THE ANALYSIS ALTHOUGH LOOKS MORE COMPLETE.

5 Conclusion

In this paper we have considered the possibility for a monopoly firm of a new product to signal quality by inducing excess demand. We have established results for a simple two period model which illustrates the strategic role of limit supplying. In both homogenous and hetergeneous consumer assumptions we show that seller induced excess demand can signal high quality under general conditions. With some additional conditions, seller-induced excess demand is a more profitable signal of high quality than using price alone. In addition, signaling high quality by inducing excess demand is always accompanied with high price that exploits all consumer surplus for individual consumers who purchase in the introductory phase. Our results provide a rationale for "limited editions", capacity constraints, or queuing, together with a high price in the introductory phase of a high-quality product provided by a monopoly firm.

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6 Appendix

Proof of Proposition 4.

Proof. Consider the binding condition (23) in least-cost separating equilibrium. Let (p_1, α) satisfy (23). We break the rest of the proof into two cases.

- **Case 1:** $\alpha \geq 1 - \frac{\theta_0 s_0 - c_0}{q_1(\theta_1 s_1 - c_0)}$.

In this case, $\theta_0 s_0 - c_0 \geq (1-\alpha) q_1(\theta_1 s_1 - c_0)$. Thus, by (23), $\alpha q_1(p_1 - c_0) = (\theta_0 s_0 - c_0)$, which implies $p_1 = c_0 + (\theta_0 s_0 - c_0)/\alpha q_1$. It follows that high-quality firm's first period profit $\alpha q_1(p_1 - c_1) = \theta_0 s_0 - c_0 - \alpha q_1(c_1 - c_0)$ is decreasing in α.

- **Case 2:** $\alpha \leq 1 - \frac{\theta_0 s_0 - c_0}{q_1(\theta_1 s_1 - c_0)}$.

In this case, $\theta_0 s_0 - c_0 \leq (1-\alpha)(\theta_1 s_1 - c_0)$. Thus, by (23), $\alpha(p_1 - c_0) + \delta(1 - \alpha) q_1(\theta_1 s_1 - c_0) = (1+\delta)(\theta_0 s_0 - c_0)$ which implies that the high-quality firm's first period profit is $\alpha q_1(p_1 - c_1) = (1+\delta)(\theta_0 s_0 - c_0) - \delta q_1(\theta_1 s_1 - c_0) + \alpha q_1(\theta_1 s_1 - c_0) - \alpha q_1 (c_1 - c_0)$. Since $\delta > (c_1 - c_0) / (\theta_1 s_1 - c_0)$ and $\theta_1 s_1 - c_0 > c_1 - c_0$, it follows that that $\widetilde{\alpha} q_1(\widetilde{p}_1 - c_1)$ is increasing in $\widetilde{\alpha}$.

In summary, the analysis in case 1 and case 2 together concludes that in any least-cost separating equilibrium it must be

$$\alpha = 1 - \frac{\theta_0 s_0 - c_0}{q_1 (\theta_1 s_1 - c_0)}, \tag{A-1}$$

and

$$p_1 = c_0 + \frac{(\theta_1 s_1 - c_0)(\theta_0 s_0 - c_0)}{q_1 (\theta_1 s_1 - c_0) - (\theta_0 s_0 - c_0)}. \tag{A-2}$$

This establishes the uniqueness. To show the existence, notice first that **A2** and (A-1) imply $0 < \alpha < 1$. Notice also $c_1 < p_1$ is automatically satisfied. By **A3**, $q_1 (\theta_1 s_1 - c_0) > 2 (\theta_0 s_0 - c_0)$ which together with (A-2) implies $p_1 < \theta_1 s_1$. Since (p_1, α) in (A-1) and (A-2) maximizes the high-quality firm's period 1 profit subject to to (17), to complete the rest of the proof it suffices to shows that (p_1, α) also satisfies (18). Notice

$$\alpha q_1(p_1 - c_1) \geq \theta_0 s_0 - c_1 \Longleftrightarrow \frac{q_1 (\theta_1 s_1 - c_0) - (\theta_0 s_0 - s_0)}{\theta_1 s_1 - c_0} \leq 1,$$

which always holds. ■